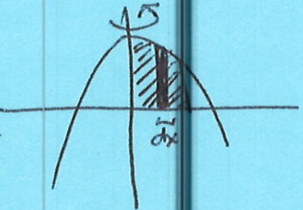
MTH 174 (ELI) Practice Exam #1 Key

1. $V = 2\pi \int_0^2 x(9-x^2)\,dx =$



$2\pi \int_0^2 9x - x^3\,dx = 2\pi\left[\frac{9}{2}x^2 - \frac{1}{4}x^4\right|_0^2$

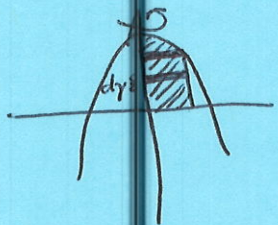
$2\pi[18-4] = 28\pi$

2. $y = 9 - x^2$

$x^2 = 9-y$

$x = \sqrt{9-y}$

$2 = \sqrt{9-y}$



$V = \pi \int_5^9 (\sqrt{9-y})^2\,dy + \pi \int_0^5 2^2\,dy =$

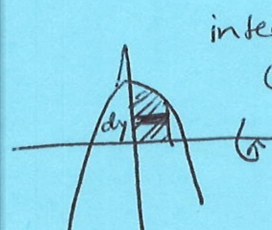
$\pi \int_5^9 9-y\,dy + \pi \int_0^5 4\,dy =$

$\pi\left[9y - \frac{1}{2}y^2\right|_5^9 + \pi\left[4y\right|_0^5 = \pi\left[81 - \frac{81}{2} - 45 + \frac{25}{2}\right] + \pi[20]$

$= 8\pi + 20\pi = 28\pi$

3. $V = 2\pi \int_5^9 y\sqrt{9-y}\,dy + 2\pi \int_0^5 y(2)\,dy$

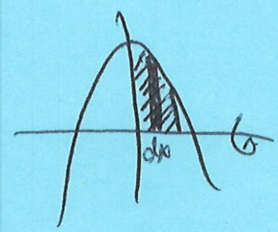
integrate w/ change of variable let $u = \sqrt{9-y}$



$= 2\pi\left[\frac{2}{5}y(9-y)^{3/2} - \frac{12}{5}(9-y)^{3/2}\right|_5^9 +$

$2\pi\, y^2\Big|_0^5 = 2\pi\left[\frac{176}{5}\right] + 2\pi[25] = \frac{602\pi}{5}$

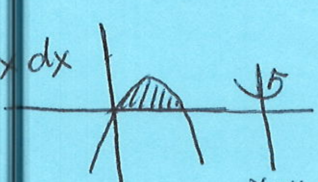
4. $V = \pi \int_0^2 (9-x^2)^2\,dx = \pi \int_0^2 81 - 18x^2 + x^4\,dx$



$= \pi\left[81x - 6x^3 + \frac{1}{5}x^5\right|_0^2 = \frac{602\pi}{5}$

5. Shell

$V = 2\pi \int_0^2 (4-x)(2x-x^2)\,dx = 2\pi \int_0^2 x^3 - 6x^2 + 8x\,dx$

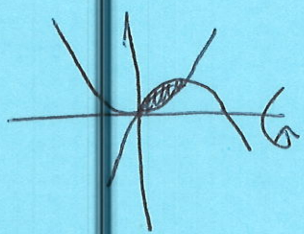


$= 2\pi\left[\frac{1}{4}x^4 - 2x^3 + 4x^2\right|_0^2 = 2\pi(4) = 8\pi$

6. $V = \pi \int_0^2 (4x - x^2)^2 - (x^2)^2 \, dx =$

$= \pi \int_0^2 16x^2 - 8x^3 \, dx = \frac{32\pi}{3}$

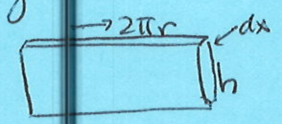
$= \pi \left[\frac{16}{3}x^3 - 2x^4 \right]_0^2$



7. The shell method is derived from the formula for a cylinder's surface area

Surface of cylinder $= 2\pi r h$

$\longrightarrow$ unwrap



the radius is distance from axis of rotation (around y-axis this is x) and the height is $f(x)$

the volume comes from the thickness of the shell

$dV = \text{Area} \cdot dx = \underline{2\pi} x \cdot f(x)\, dx$

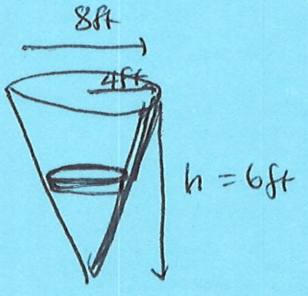
π is there because the length is determined by circumference of circle on top of cylinder

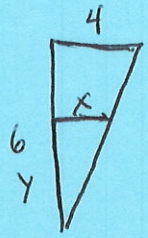
8. $y = 9 - x^2$
$y' = -2x$

$S = \int_0^2 \sqrt{1 + (-2x)^2}\, dx = \int_0^2 \sqrt{1 + 4x^2}\, dx \approx 4.65$

requires integration w/ trig sub. use calculator to evaluate

9.





$\frac{x}{y} = \frac{4}{6} \rightarrow x = \frac{2}{3}y$

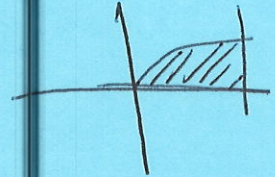
$V = \pi (\frac{2}{3}y)^2 dy$

water density − 62.4 lbs/ft^3

$W = \int_0^6 62.4\pi (\frac{4}{9}) y^2 dy \cdot (6-y) = \frac{62.4 \cdot 4\pi}{9} \int_0^6 6y^2 - y^3 \, dy = \frac{62.4 \cdot 4\pi}{9} \left[2y^3 - \frac{1}{4}y^4 \right]_0^6$

$\frac{62.4 \cdot 4\pi}{9}(108) = 2995.2\pi$

10. $M = \int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}\,dx = \frac{2}{3}x^{3/2}\Big|_0^4 =$



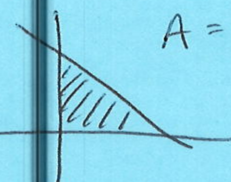
$\frac{2}{3}(8) = \frac{16}{3}$

$M_y = \int_0^4 x\sqrt{x}\,dx = \int_0^4 x^{3/2}\,dx = \frac{2}{5}x^{5/2}\Big|_0^4 = \frac{2}{5}(32) = \frac{64}{5}$

$M_x = \int_0^4 (\sqrt{x})^2\,dy = \int_0^4 x\,dx = \frac{1}{2}x^2\Big|_0^4 = 8$

$\bar{x} = \frac{\cancel{64}^{\,4}}{5}\cdot\frac{3}{\cancel{16}} = \frac{12}{5}$ $\quad \bar{y} = \frac{8}{\frac{16}{3}} = \frac{\cancel{8}}{1}\cdot\frac{3}{\cancel{16}_2} = \frac{3}{2}$ $\quad \left(\frac{12}{5}, \frac{3}{2}\right)$

11. $\int_0^4 (x)(-x+4)\,dx = \int_0^4 -x^2+4x\,dx =$ $\quad A = \frac{1}{2}(4)(4) = 8$



$-\frac{1}{3}x^3 + 2x^2\Big|_0^4 = \frac{32}{3}$ $\quad \bar{x} = \frac{\cancel{32}^{\,4}}{3}\cdot\frac{1}{\cancel{8}} = \frac{4}{3}$

$\frac{1}{2}\int_0^4 (-x+4)^2\,dx = \frac{1}{2}\int_0^4 x^2-8x+16\,dx = \frac{1}{2}\Big[\frac{1}{3}x^3+4x^2+16x\Big|_0^4 = \frac{1}{2}\cdot\frac{64}{3}$ $\quad \bar{y} = \frac{\cancel{64}^{\,8}}{3}\cdot\frac{1}{\cancel{8}} = \frac{8}{3}$

$\frac{8}{3}\cdot\frac{1}{2} = \frac{4}{3}$

$(\bar{x}, \bar{y}) = \left(\frac{4}{3}, \frac{4}{3}\right)$

12. $x = \frac{1}{3}(y^2+2)^{3/2}$ $\quad \frac{dx}{dy} = \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}(y^2+2)^{1/2}\cdot\cancel{2}y = y\sqrt{y^2+2}$

$S = \int_0^4 \sqrt{1+y^2(y^2+2)}\,dy = \int_0^4 \sqrt{1+y^4+2y^2}\,dy = \int_0^4 \sqrt{(y^2+1)^2}\,dy =$

y^4+2y^2+1

$\int_0^4 y^2+1\,dy = \frac{1}{3}y^3+y\Big|_0^4 = \frac{64}{3}+4 = \frac{76}{3}$

13. $\lim_{x\to 0^+} \frac{\ln x}{\cot x} = \frac{-\infty}{\infty} \to \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc^2 x} = \lim_{x\to 0^+} \frac{-\sin^2 x}{x} = \lim_{x\to 0^+} \frac{-2\sin x\cos x}{1} = 0$

14. $M = \int_0^9 1+3\sqrt{x}\,dx = x + \cancel{3}\cdot\frac{2}{\cancel{3}}x^{3/2}\Big|_0^9 = 9 + 2(27) = 63$

$x^{1/2}$

$M_y = \int_0^9 x(1+3\sqrt{x})\,dx = \int_0^9 x+3x^{3/2}\,dx = \frac{1}{2}x^2 + 3\cdot\frac{2}{5}x^{5/2}\Big|_0^9 = \frac{81}{2} + \frac{6}{5}(243) =$

$\frac{3321}{10}$

$\bar{x} = \frac{3321}{10}\cdot\frac{1}{63} = \frac{369}{70} \approx 5.27$

15. $\lim_{x \to 0^+} (1+x)^{\csc x} = L$ $\quad\quad$ $\lim_{x \to 0^+} \ln\left[(1+x)^{\csc x}\right] = \ln L$

$\lim_{x \to 0^+} \csc x \ln(1+x) = \lim_{x \to 0^+} \frac{\ln(1+x)}{\sin x} = \lim_{x \to 0^+} \frac{\frac{1}{1+x}}{\cos x} = 1$

$\ln L = 1 \Rightarrow L = e^1 = e$

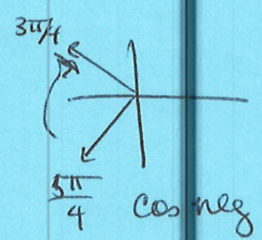
16. a. $f'(x) = \frac{1}{1+x^2}$

b. $g'(x) = \frac{1}{\sqrt{1-(3e^x+1)^2}} \cdot 3e^x = \frac{3e^x}{\sqrt{1-(3e^x+1)^2}}$

c. $h'(x) = \frac{-1}{\sqrt{1-(\ln(\tan x))^2}} \cdot \frac{1}{\tan x} \cdot \sec^2 x$

d. $F'(x) = 3(\sec^{-1} x^2)^2 \cdot \frac{1}{x^2\sqrt{(x^2)^2-1}} \cdot 2x$

17. a. $\frac{3\pi}{4}$



b. $\sqrt{1-x^2}$