MTH 174, Practice Exam #1, Summer 2018 Name ______________________________________

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Find the volume of the surface of revolution bounded by $y = 9 - x^2$ on $[0,2]$ revolved around the y-axis.
   a. Sketch the graph of the region.
   b. Using the shell method, set up the integral and label the differential on the graph.

2. Find the volume of the surface of revolution bounded by $y = 9 - x^2$ on $[0,2]$ revolved around the y-axis.
   c. Sketch the graph of the region.
   d. Using the disk method, set up the integral and label the differential on the graph.

3. Find the volume of the surface of revolution bounded by $y = 9 - x^2$ on $[0,2]$ revolved around the x-axis.
   e. Sketch the graph of the region.
   f. Using the shell method, set up the integral and label the differential on the graph.

4. Find the volume of the surface of revolution bounded by $y = 9 - x^2$ on $[0,2]$ revolved around the x-axis.
   g. Sketch the graph of the region.
   h. Using the disk method, set up the integral and label the differential on the graph.

5. Find the volume of revolution bounded by $y = 2x - x^2, y = 0$, around the line $x = 4$.

6. Find the volume of revolution bounded by $y = x^2, y = 4x - x^2$ around the x-axis.

7. Explain the derivation of the shell method formula. In particular, explain the inclusion of π in the formula, and the meaning of the dy or dx in the formula.

8. Find the arc length of the curve $y = 9 - x^2$ on the interval $[0,2]$.

9. Find the work done in emptying a conical tank by pumping the water over the top edge. The tank is 8 ft. across and 6 ft. high. The tank is initially full.

10. Find the center of mass for the planar region bounded by $y = \sqrt{x}, y = 0, x = 4$.

11. Find the center of mass for the region bounded by $y = -x + 4$ and the two coordinate planes. [Hint: This is a triangular region. Find the area of the region geometrically, and recall: $\bar{x} = \frac{\int_a^b xf(x)dx}{Area}, \bar{y} = \frac{\frac{1}{2}\int_a^b f^2(x)dx}{Area}$.]

12. Find the arc length for $x = \frac{1}{3}(y^2 + 2)^{3/2}$ on $[0,4]$. Set up the integral then and evaluate it.

13. Evaluate the limit $\lim_{x \to 0^+} \frac{\ln x}{\cot x}$.

14. A thin rod has a density that varies according to the function $\rho(x) = 1 + 3\sqrt{x}$. Find the center of mass of the rod if the left end of the rod is placed at $x = 0$, and ends at $x = 9$.

15. Evaluate $\lim_{x \to 0^+} (1 + x)^{\csc x}$.

16. Find the derivative of each of the following:
    a. $f(x) = \arctan x$

    b. $g(x) = \sin^{-1}(3e^x + 1)$

    c. $h(x) = \arccos(\ln(\tan x))$

    d. $F(x) = (\sec^{-1} x^2)^3$

17. Simplify each of the following expressions.
    a. $\cos^{-1}\left(\cos \frac{5\pi}{4}\right)$

    b. $\cos(sin^{-1} x)$