Math 240, Quiz #9, Summer 2017 Name KEY

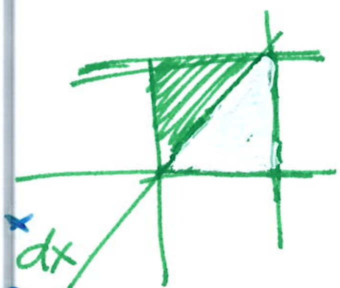
**Instructions**: Show all work. Use exact answers unless specifically asked to round.

1. Change the order of integration and find the value of the integral.

$$\int_0^9 \int_y^9 e^{-x^2}\,dxdy$$

$x=9$

$x=y$



$$\int_0^9 \int_0^x e^{-x^2}\,dydx = \int_0^9 ye^{-x^2}\Big|_0^x dx$$

$$\int_0^9 xe^{-x^2}\,dx = -\frac{1}{2}e^{-x^2}\Big|_0^9 = -\frac{1}{2}\left[e^{-81}-e^{0}\right] =$$

$$\boxed{\frac{1}{2}-\frac{1}{2e^{81}}}$$

2. Convert the integral from rectangular to polar to integrate. Find the value of the integral. Sketch the region.

$x^2+y^2=8$

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \sin\sqrt{x^2+y^2}\,dydx$$

$x=y$

$$\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{8}} r\sin r\,drd\theta =$$

$r=\sqrt{8}$

$u=r \quad dv=\sin r$

$du=dr \quad v=-\cos r$

$$\int_{\pi/4}^{\pi/2}\left[-r\cos r\Big|_0^{\sqrt{8}}+\int_0^{\sqrt{8}}\cos r\,dr\right]d\theta = \int_{\pi/4}^{\pi/2}\left[-r\cos r+\sin r\Big|_0^{\sqrt{8}}\right]d\theta$$

$$= \int_{\pi/4}^{\pi/2} -\sqrt{8}\cos\sqrt{8}+\sin\sqrt{8}\,d\theta = \left(\sin\sqrt{8}-\sqrt{8}\cos\sqrt{8}\right)\theta\Big|_{\pi/4}^{\pi/2} =$$

$$\boxed{\frac{\pi}{4}\left[\sin\sqrt{8}-\sqrt{8}\cos\sqrt{8}\right]}$$