Math 240, Quiz #7, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the total differential of $g(x,y,z)=x^2yz^2+\sin yz$. Evaluate $g(1,2,0)$ to estimate the value of $g(1.05,2.1,-0.01)$.

$dg \approx g_x\,dx + g_y\,dy + g_z\,dz$

$g_x = 2xyz^2$

$g_y = x^2z^2 + z\cos yz$

$g_z = 2x^2yz + y\cos yz$

$dx = .05$

$dy = .1$

$dz = -.01$

$g_x(1,2,0) = 0$

$g_y(1,2,0) = 0$

$g_z(1,2,0) = 2$

$dg = (0)(.05) + 0(.1) + 2(-.01) = \boxed{-.02}$

2. Find all critical points of the function $f(x,y)=x^2+6xy+10y^2-4y+4$ and use the second partials test to determine if the point is a maximum, a minimum, or a saddle point (or cannot be determined).

$f_x = 2x + 6y = 0 \Rightarrow 2x = -6y$

$x = -3y$

$x = -6$

$f_y = 6x + 20y - 4 = 0$

$6(-3y) + 20y - 4 = 0$

$-18y + 20y - 4 = 0$

$2y = 4$

$y = 2$

$(-6, 2)$

$f_{xx} = 2$

$f_{yy} = 20$

$f_{xy} = 6$

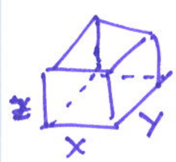
$D = 2(20) - 6^2 =$

$40 - 36 = 4 > 0$

max or min $\Rightarrow$ concave up $\cup$ $\Rightarrow$ $\boxed{\text{min}}$

3. Find the maximum and minimum volumes of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm.

$V = xyz$



$SA = 2xy + 2yz + 2xz = 1500$

$xy + yz + xz = 750$

$EL = 4z + 4x + 4y = 200$

$x + y + z = 50$

$\nabla V = \lambda \nabla g + \mu \nabla h$

$V = xyz$

$g = xy + yz + xz - 750$

$h = x + y + z - 50$

$yz = \lambda(y+z) + \mu$

$xz = \lambda(x+z) + \mu$

$xy = \lambda(y+x) + \mu$

$$\left.\begin{aligned} \mu &= yz - \lambda y - \lambda z \\ \mu &= xz - \lambda x - \lambda z \\ \mu &= xy - \lambda y - \lambda x \end{aligned}\right\} \rightarrow \begin{aligned} yz - \lambda y - \cancel{\lambda z} &= xz - \lambda x - \cancel{\lambda z} \\ xz - \cancel{\lambda x} - \lambda z &= xy - \lambda y - \cancel{\lambda x} \end{aligned}$$

$\Downarrow$

$yz - xz = \lambda y - \lambda x$

$z(y-x) = \lambda(y-x)$

$z = \lambda \leftarrow$ or $\rightarrow y = x$

$xz - \lambda z = xy - \lambda y$

$xz - xy = \lambda z - \lambda y$

$x(z-y) = \lambda(z-y)$

$x = \lambda \leftarrow$ or $\rightarrow z = y$

any 2 can be alike w/ 3rd diff. Symmetric w/ respect to orientations.

$y = x$

$x^2 + xz + xz - 750 = 0$

$x^2 + 2xz = 750$

$2x + z = 50$

$z = 50 - 2x$

$x^2 + 2x(50-2x) = 750$

$x^2 + 100x - 4x^2 = 750$

$-3x^2 + 100x - 750 = 0$

$3x^2 - 100x + 750 = 0$

$x = \dfrac{100 \pm \sqrt{10{,}000 - 9000}}{6} = \dfrac{100 \pm \overset{10\sqrt{10}}{\sqrt{1000}}}{6} = \dfrac{50 \pm 5\sqrt{10}}{3}$ ≈ 21.94, ≈ 11.40

$x = y \qquad z = 50 - \left(\dfrac{100 \pm \sqrt{1000}}{3}\right)$

one is min, one is max

$V = 2945.9$

$(21.94, 21.94, 6.12)$ max

$(11.40, 11.40, 27.2)$ min

$V = 157.16$