Math 240, Quiz #13, Summer 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. For the surface $\vec{r}(u,v) = (1-u)(3+\cos v)\hat{\imath} + (1-u)(3+\cos v)\hat{\jmath} + (3u + (1-u)\sin v)\hat{k}$,
   a. Find the first partial derivatives

$\vec{r}_u = -(3+\cos v)\hat{\imath} - (3+\cos v)\hat{\jmath} + (3-\sin v)\hat{k}$

$\vec{r}_v = (1-u)(-\sin v)\hat{\imath} + (1-u)(-\sin v)\hat{\jmath} + (1-u)\cos v\,\hat{k}$

   b. Find the equation of the tangent plane at $u = 0, v = 0$.

$\vec{r}(0,0) = (1)(4)\hat{\imath} + (1)(4)\hat{\jmath} + 0\hat{k}$ $\quad (4,4,0)$

$\vec{r}_u(0,0) = -4\hat{\imath} - 4\hat{\jmath} + 3\hat{k}$ $\quad \vec{r}_v(0,0) = 0\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & -4 & 3 \\ 0 & 0 & 1 \end{vmatrix} = (-4-0)\hat{\imath} - (-4-0)\hat{\jmath} + 0\hat{k} \quad \langle -4, 4, 0 \rangle$$

$$-4(x-4) + 4(y-4) = 0$$

2. Find the divergence and the curl of $\vec{F}(x,y,z) = (x^2y - z)\hat{\imath} + (x\sin(y) + yz^3)\hat{\jmath} + (4xze^y)\hat{k}$.

$\vec{\nabla}\cdot\vec{F} = 2xy + (x\cos y + z^3) + 4xe^y$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y - z & x\sin y + yz^3 & 4xze^y \end{vmatrix} =$$

$$(4xze^y - 3yz^2)\hat{\imath} - (4ze^y + 1)\hat{\jmath} + (\sin y - x^2)\hat{k}$$