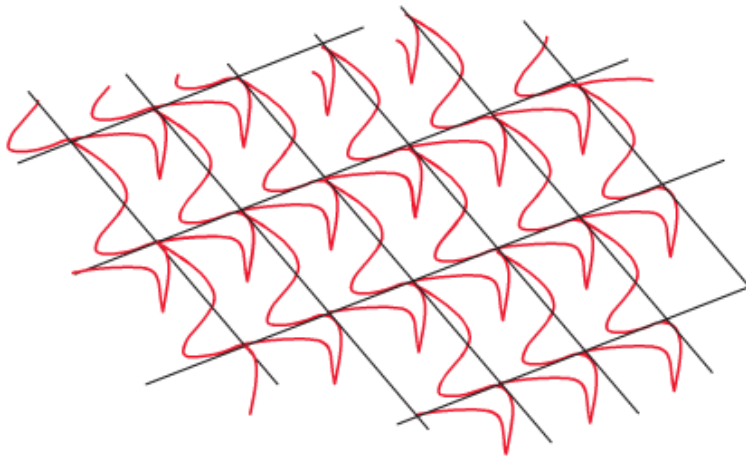# Tessellation Assignment Ideas

M. C. Escher's birds from a grid of parallelograms:

The concept of a translation, discussed in Section 11.1, helps us understand how the parallelogram has been modified to become the bird-shaped motif of the tiling. First, imagine replacing the upper edge of the parallelogram with the *V*-shape separating the wings. The *V*-shape is then translated to replace the opposite edge of the parallelogram. Similarly, one of the two remaining straight edges of the parallelogram is modified to form the leading edge of the forward wing, and this curve is then translated to replace the opposite edge of the parallelogram. Finally, the outline is filled in with details such as feathers and an eye to complete the bird motif. The steps modifying the paral-


Becomes…

From: Long, DeTemple, & Millman, *Mathematics Reasoning for Elementary Teachers*, 6th Ed., pp. 643-649.

LET'S PRACTICE!

Materials Needed

1. Note cards, 3" × 5" (or other card stock)
2. Scissors
2. Pencils and colored markers
4. Blank sheets of paper

Directions

Step 1. Cut a small (say, $2\frac{1}{2}$"-by-3") rectangle from a note card or card stock.

Step 2. Make an irregular curve joining the corners of one side of the rectangle.

Step 3. Cut out the curve. Translate the cutout piece to the opposite side and tape it in place.

Step 4. Repeat steps 2 and 3 for the remaining two parallel sides of the rectangle, as shown.

Step 5. Do an "inkblot" test. Is your shape a frog, a bird, a face, a _____? Brainstorm with a partner. It may help to rotate your shape or flip it over. Add eyes, mouth, nose, ears, feet, beaks, horns, clothing, scales, fur, and other imaginative details to make your tiling template recognizable and interesting.
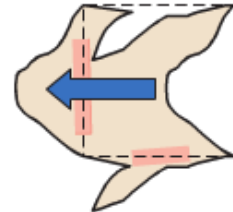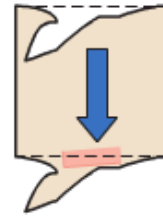
Step 6. Trace around the template on a blank sheet of paper. Translate and trace again, repeating to create at least three rows and three columns of your interlocking tiles to create your Escher-like tiling. Use colored markers to fill in the details, and color adjacent tiles with different colors.
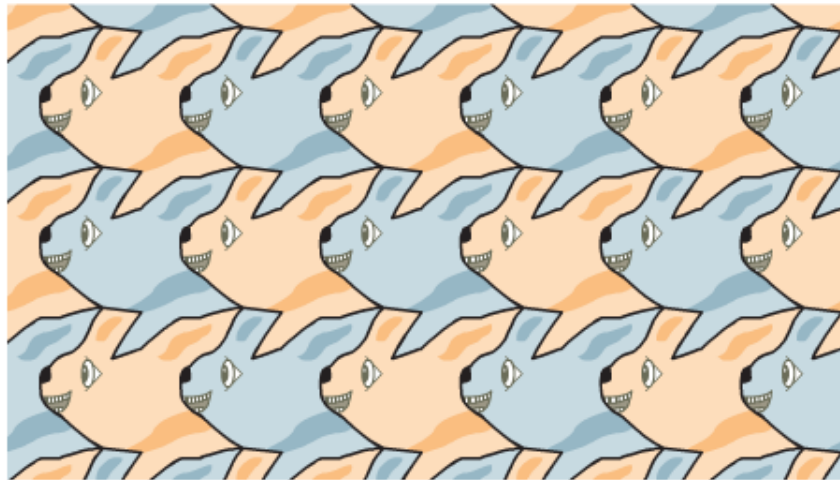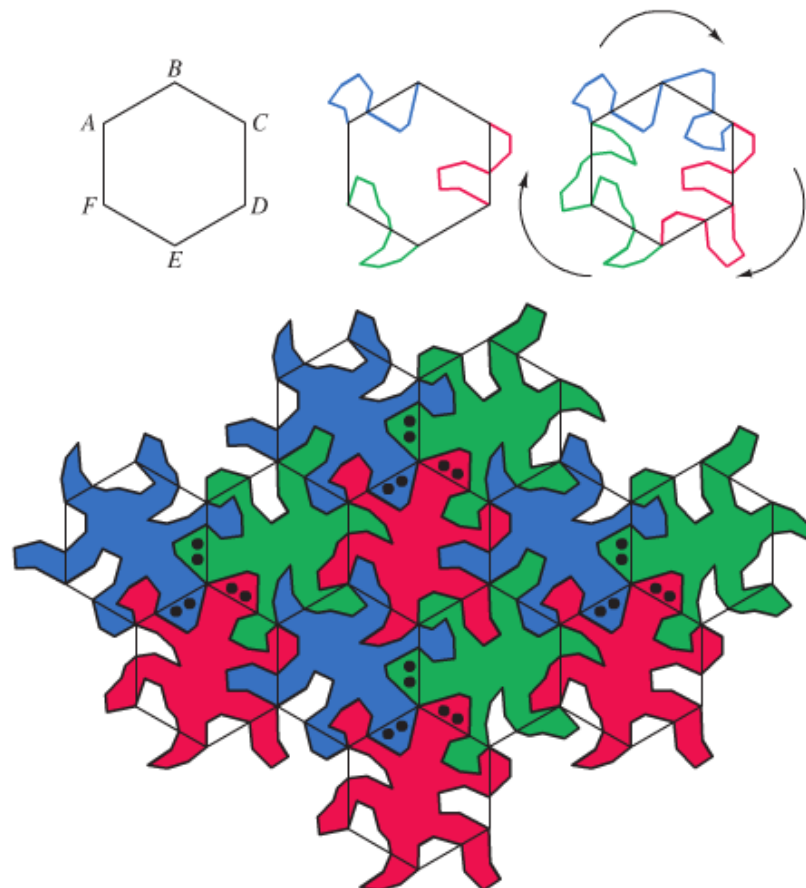
Extensions

Instead of a rectangle, start with any parallelogram (or any hexgon with opposite sides parallel and congruent) and follow the directions just given. It is also possible to adapt this method to create templates based on other tilings of the plane. Several suggestions are described in problems 20 through 23 of Problem Set 11.3.

MORE IDEAS TO HELP GET YOU GOING:

Similar procedures will transform any polygonal tiling that can be produced by translations to an Escher-like tiling. For example, sixth-grade teacher Nancy Putnam used translations to modify each of the three pairs of opposite parallel congruent sides of a hexagon. Her whale tiling, starting with hexagon *ABCDEF*, is shown in Figure 11.29.
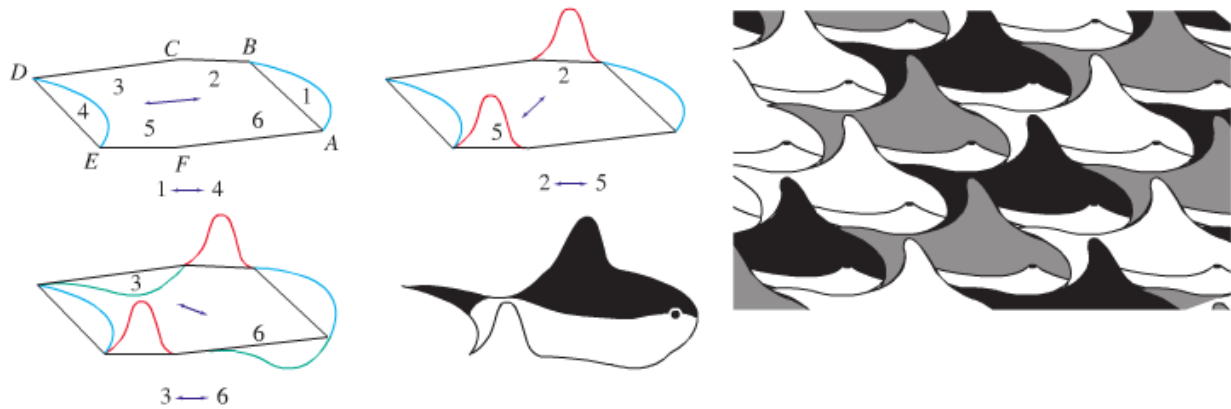
Rotations can also be used to create **Escher-like tiles** with interesting symmetries. Figure 11.30 shows how a lizard tile can be created by modifying a regular hexagon *ABCDEF*. Side $\overline{AB}$ is first modified and then rotated about vertex *B* to modify side $\overline{BC}$. The remaining two pairs of adjacent sides are also modified by rotations, resulting in the outline of the lizard tile shown in the figure, along with its tiling.

From: Long, DeTemple, & Millman, *Mathematics Reasoning for Elementary Teachers*, 6th Ed., pp. 643-649.

20. Construct a paper hexagonal tile with each pair of opposite sides parallel and congruent. The template can be cut from a note card following the method described in problem 9. Make cutouts on three adjacent sides. Translate each cutout to the opposite side and tape along the corresponding edges to form a template. (See the instructions in the Cooperative Investigation box "Creating an Escher-like Design.")
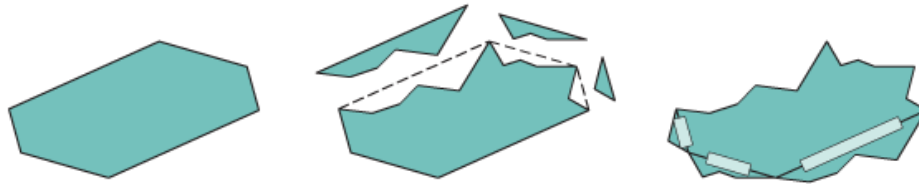
Create a design with your template, adding details such as eyes, mouths, and so forth, to give added interest to the design.

21. Cut an accurate square from a note card. Make cutouts on opposite sides. Rotate each cutout 90° and tape as shown. Use the paper template to create an Escher-like tiling. You will need to use 90° rotations to produce the tiling.
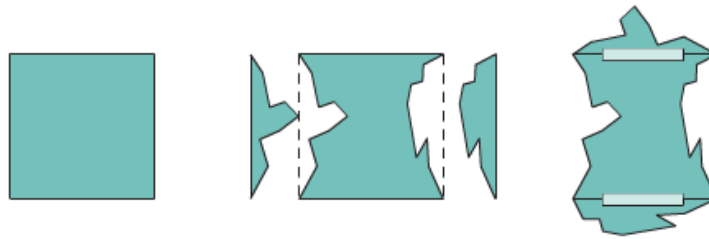
22. Cut an arbitrary triangle from a note card, and lightly fold (do not make a heavy crease) one vertex to another to determine the midpoint of the side between the vertices. The side can be modified by making a cutout on one side of the midpoint, rotating the cutout 180° about the midpoint, and taping it in place. The steps to modify one side of a triangle are as follows:

Midpoint of side

Modify the remaining two sides of the triangle in a similar manner and use the resulting template to create an Escher-like tiling. You will need to use 180° rotations to produce the tiling.