MAT 222

Chapter 8: Things To Know

<u>Section 8.1 Rigid Transformations</u>

Objectives	**Vocabulary**
1. Identify Rigid Transformations or Isometries. 2. Name Images and Corresponding Parts.	• transformation • preimage • image • isometry • rigid transformation

A ______________________________________ of a figure in a plane is a change of the position, shape, or size of the figure.

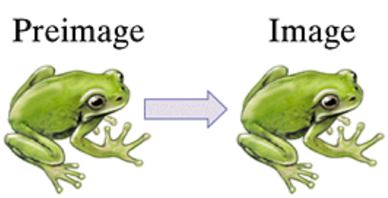
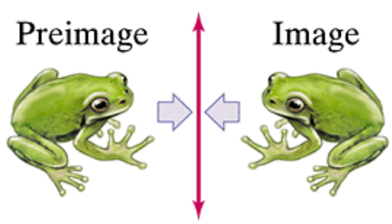
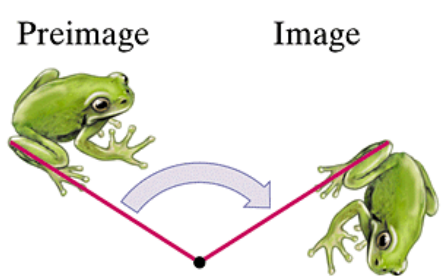
In a transformation, the original figure is the ____________________________.

The resulting figure is the ____________________________.

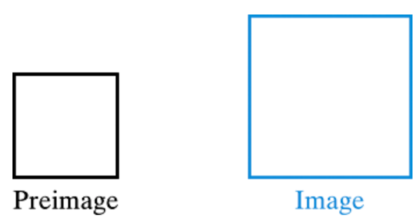
An ___________________________ is a transformation in which the preimage and the image are congruent.

This type of transformation is also called a ____ because the original shape and size of the figure do not change.

The three types of rigid transformations we will study in this section are:

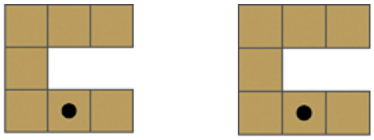
		

Example Identifying an Isometry
Does the transformation to the right appear to be an isometry? Explain.



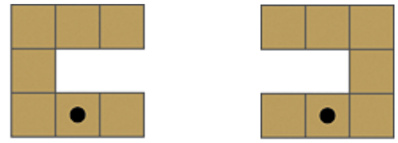
Example Identifying Single Rigid Transformations
Identify the single transformation from the preimage to each image.

a.



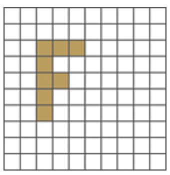
Preimage Image

b.



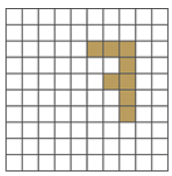
Preimage Image

Example Identifying Single Rigid Transformations
Identify the single transformation from the preimage to each individual image.



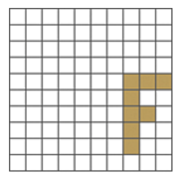
Preimage

a.



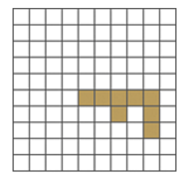
Image

b.



Image

c.



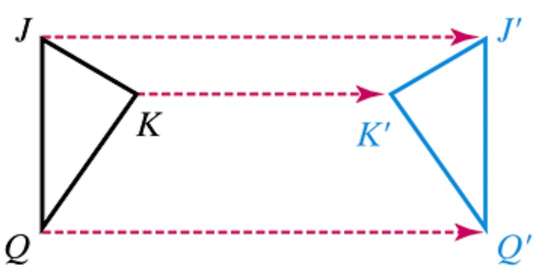
Image

Naming Images

A transformation maps (or moves) a figure to its image and may be described with arrow notation. (→)

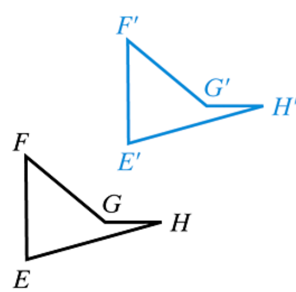
Prime notation (‘) is sometimes used to identify image points.

Name three transformations in the diagram below using arrow and prime notation.



- *K*
- *J*
- *Q*

Example Naming Images and Corresponding Parts
In the diagram, *EFGH* maps to *E’F’G’H’*.



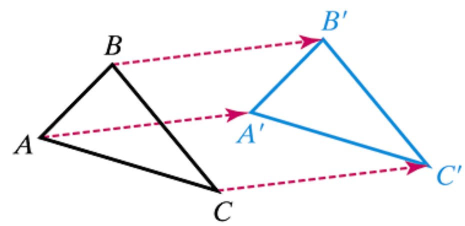
a. What are the images of point *F* and point *H*?

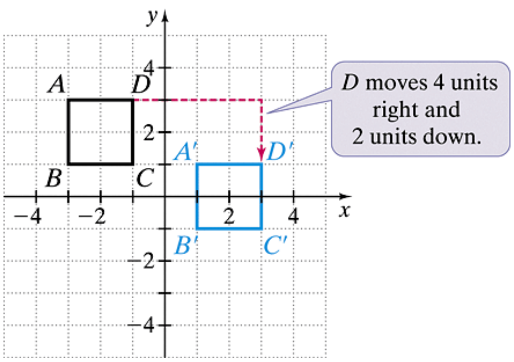
b. What are the pairs of congruent corresponding sides?

Section 8.2 Translations

Objectives	Vocabulary
1. Find Translation Images of Figures.	• translation • composition of transformations

A ______________________ is a transformation that maps all points of a figure the same distance in the same direction. This type of transformation is an isometry.





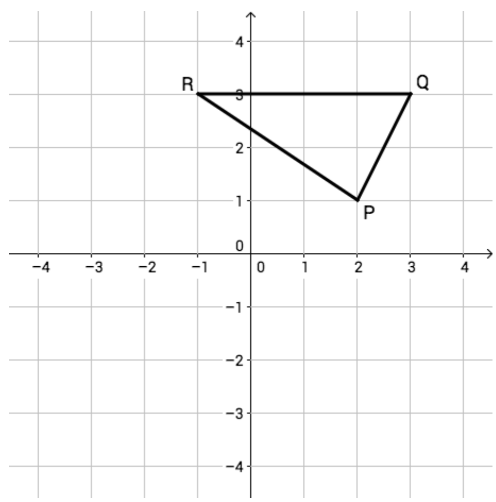
In the example to the left, each point of the square moves 4 units right and 2 units down. Using ordered pair notation, we say that each (x, y) point in the original figure is mapped to (x', y'), where $x' = x + 4$ and $y' = y - 2$.

We will use mapping notation to write this *translation rule* as

$(x, y) \rightarrow$ ______________________.

Example Finding the Image of a Translation

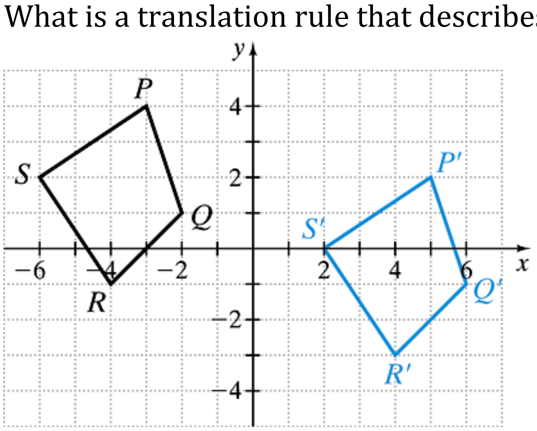
a. Find the image of each vertex of ΔPQR for the translation $(x, y) \rightarrow (x - 2, y - 5)$.



b. Graph the image of ΔPQR.

Example Writing a Rule to Describe a Translation

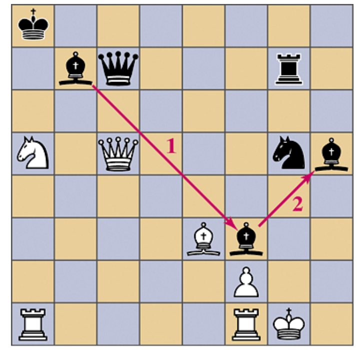
What is a translation rule that describes the translation $PQRS \rightarrow P'Q'R'S'$?



A ____ is a combination of two or more transformations. In a composition, we perform each transformation on the image of the preceding transformation.

Example Composing Translations

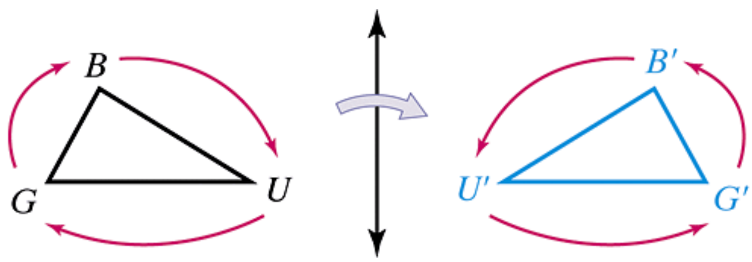
The diagram at the right shows two moves of the black bishop in a chess game. Where is the bishop in relation to its original position?



Section 8.3 Reflections

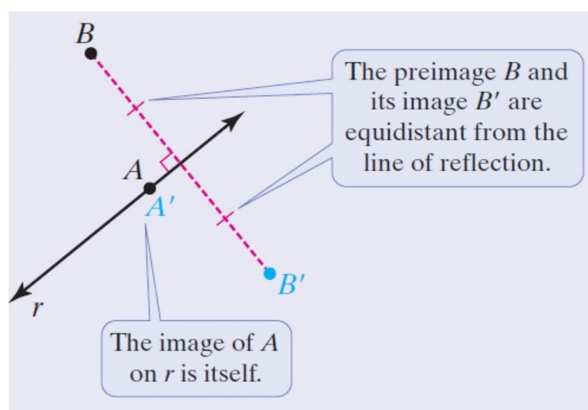
Objectives	**Vocabulary**
1. Find the Reflection Images of Figures. 2. Identify Line Symmetry.	• reflection • line of reflection • line symmetry • reflectional symmetry • line of symmetry

When a figure flips across a line, the preimage and its image are congruent and have *opposite orientations.*



The size and shape of a geometric figure stay the same when we flip the figure across a line.

________________________________ across a line r, called the ______________________________________,
is a transformation with these two properties:



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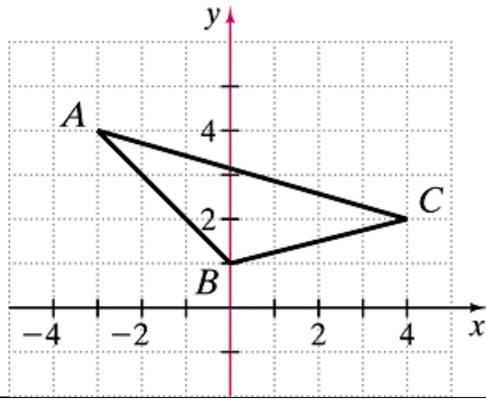
A reflection across a line is an isometry.

Example Reflecting a Point Across a Line
Multiple Choice If point $P(3,4)$ is reflected across the line $y = 1$, what are the coordinates of its reflection image?

a. $(3, -4)$ b. $(0,4)$ c. $(3, -2)$ d. $(-3, -2)$

Example Graphing a Reflection Image

Coordinate Geometry Graph points $A(-3,4)$, $B(0,1)$, and $C(4,2)$. What is the image of ΔABC reflected across the y-axis?



Example Minimizing a Distance

Beginning from a single point on Summit Trail, a hiking club will build two trails–one trail to the Overlook and one trail to Balance Rock. Working under a tight budget, the club members want to minimize the total length of the two trails. How can we find the point on Summit Trail where the two new trails should start? Assume the trails will be straight and that they will cover similar terrain.

A plane figure has ____

if the figure on one side of the line is the reflection of the figure on the other side of the line. The line of reflection is called a ______________________________________. It divides a plane figure into congruent halves.

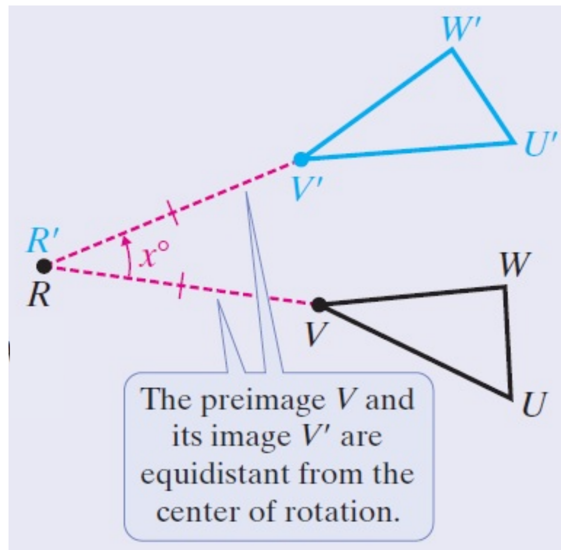
Example Identifying Lines of Symmetry

How many lines of symmetry does a regular hexagon have?

Section 8.4 Rotations

Objectives	**Vocabulary**
1. Draw and Identify Rotation Images of Figures. 2. Identify Rotational Symmetry.	• rotation • center of rotation • angle of rotation • center of a regular polygon • symmetry • rotational symmetry

A _________________________ of $x°$ about a point R, called the ____ , is a transformation with these two properties:



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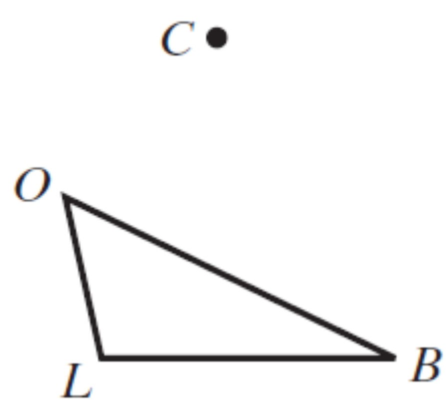
•

The positive number of degrees a figure rotates is the

____.

A rotation about a point is an isometry.

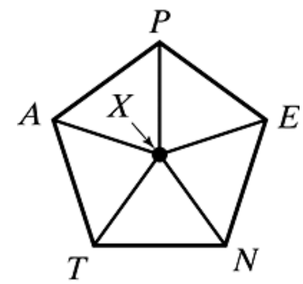
Example Drawing a Rotation Image
What is the image of ΔLOB for a 100 degree rotation about C?



The ________________________________ of a regular polygon is the point that is equidistant from its vertices.

The center and the vertices of a regular n-gon determine n ____.
We can use this fact to find rotation images of regular polygons.

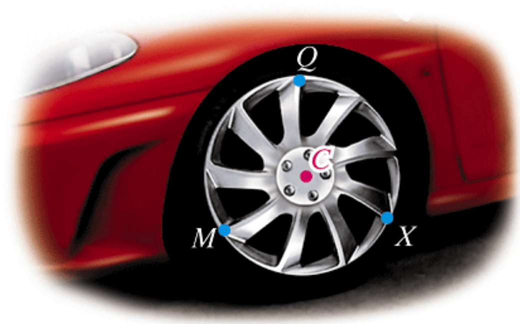
Example Identifying a Rotation Image
Point X is the center of a regular pentagon $PENTA$. What is the image of the given point of segment for the given rotation?



a. 72° rotation of T about X.

b. 216° rotation of segment TN about X

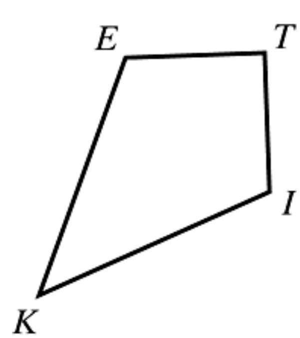
Example Finding an Angle of Revolution
Hubcaps of car wheels often have interesting designs that involve rotation. What is the angle of rotation, in degrees, about C that maps Q to X?



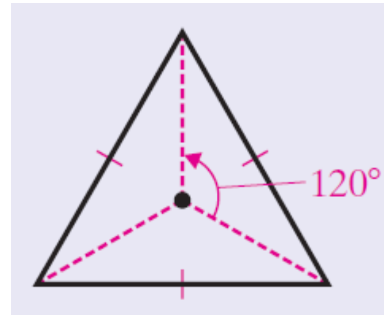
A composition of rotations about the same point is itself a rotation about that point. To sketch the image,

____.

Example Finding a Composition of Rotations
What is the image of $KITE$ for a composition of a 30° rotation and a 60° rotation, both about point K?

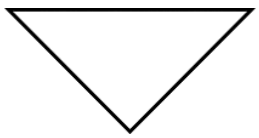


A figure has ____
if there is a rotation of 180° or less for which the figure is its own image. The angle of rotation for rotational symmetry is the smallest angle needed for the figure to rotate onto itself.



Example Identifying Rotational Symmetry

Does the figure have rotational symmetry? If so, what is the angle of rotation?

a. 

b. 

Section 8.5 Dilations

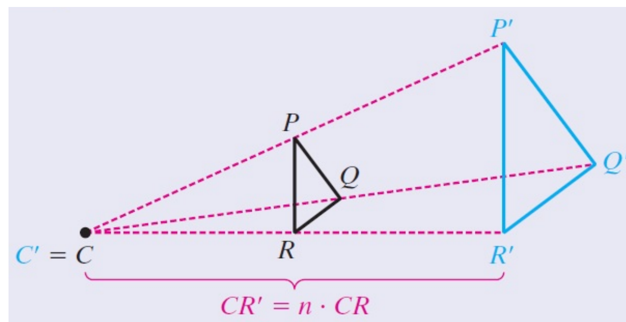
Objectives	**Vocabulary**
1. Understand Dilation Images of Figures.	• dilation • center of dilation • scale factor of a dilation • enlargement • reduction

We can use a ____ to make a larger or smaller copy of a figure that is also similar to the original figure.

A ________________________ with ________________________ C, and ________________________ $n > 0, n \neq 1$, is a transformation with these two properties:

•

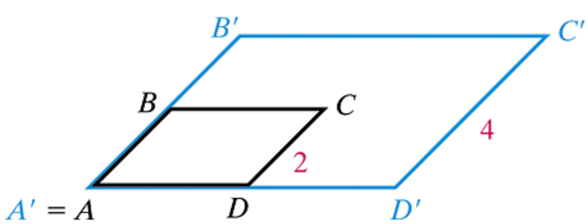
•

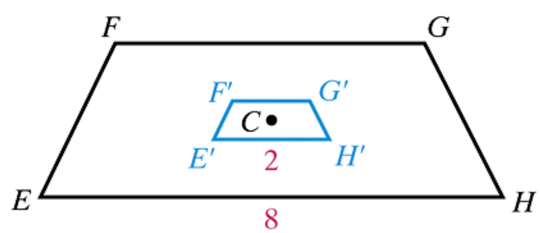


The image of a dilation is similar to its preimage.

A dilation is an ______________________________ if the scale factor is greater than 1.

The dilation is a ______________________________ if the scale factor is between 0 and 1.





Example Finding a Scale Factor

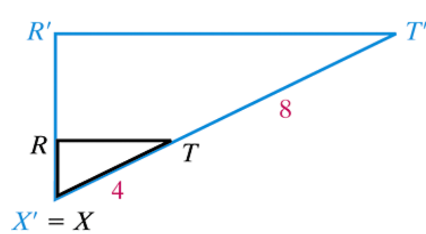
Multiple Choice $\Delta X'T'R'$ is a dilation image of ΔXTR. The center of dilation is X. Is the dilation an enlargement or a reduction? What is the scale factor of the dilation?

a. enlargement; scale factor 2

b. enlargement; scale factor 3

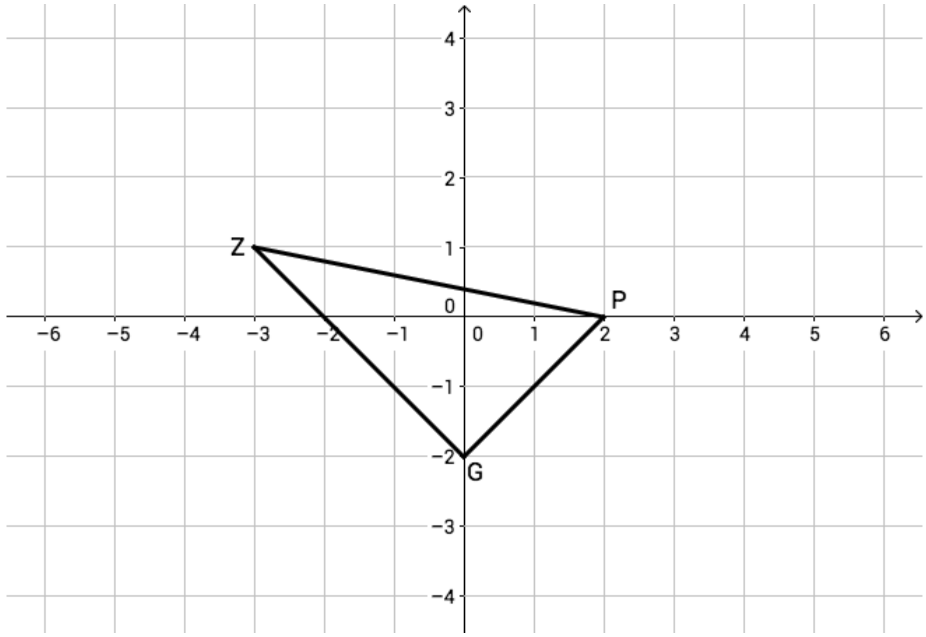
c. reduction; scale factor 1/3

d. reduction; scale factor 3



Example Finding a Dilation Image

What are the images of the vertices of ΔPZG for a dilation with center (0,0) and scale factor 2? Graph the image of ΔPZG.



Example Using a Scale Factor to Find a Length

A magnifying glass shows you an image of an object that is 7 times the object's actual size. So the scale factor of the enlargement is 7. The photo shows an apple seed under this magnifying glass. What is the actual length of the apple seed?

Section 8.6 Composition of Reflections

Objectives	Vocabulary
1. Find Compositions of Reflections, Including Glide Reflections. 2. Classify Isometries.	• glide reflection

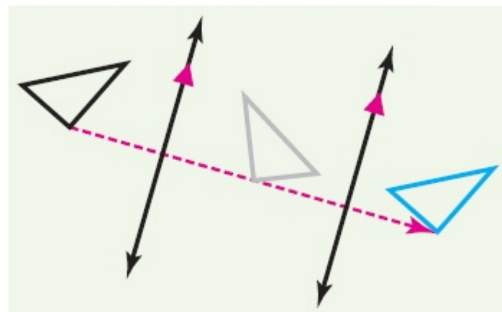
Any isometry can be expressed as a composition of reflections.

If two figures in a plane are congruent, we can map one onto the other using a composition of reflections.

Theorem A translation of a rotation is a composition of two reflections.

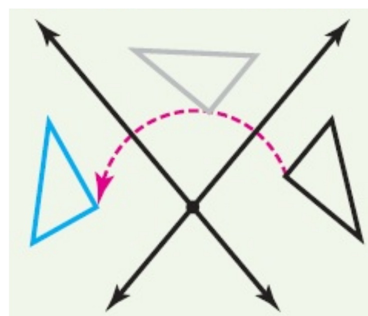
Theorem
A composition of reflections across two parallel lines is a

___________________________________.

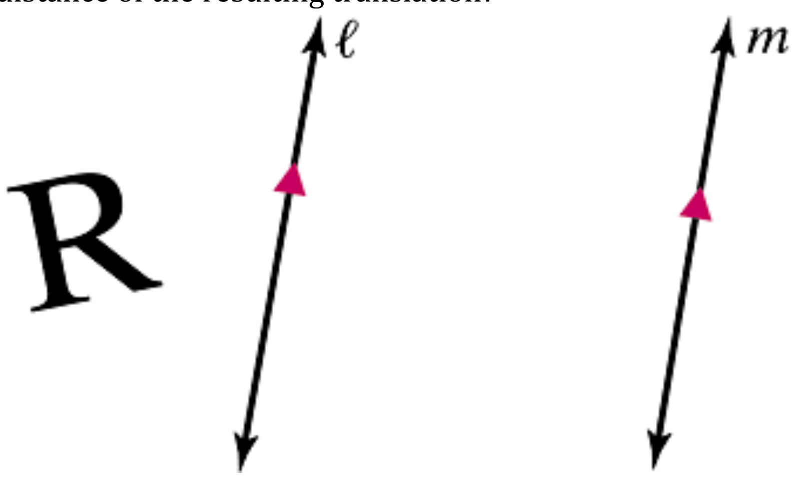


Theorem
A composition of reflections across two intersecting lines is a

___________________________________.

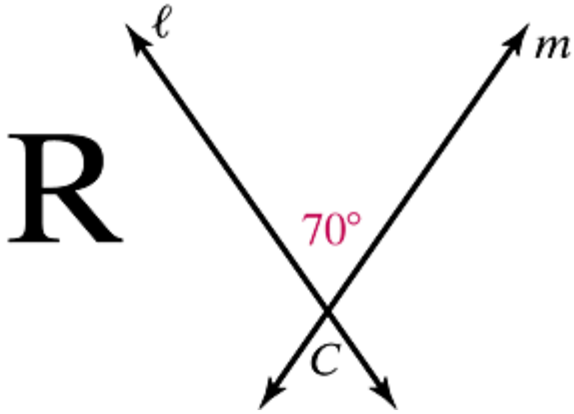


Example Composing Reflections Across Parallel Lines
What is the image of R reflected first across line ℓ and then across line *m*? What are the direction and distance of the resulting translation?



Example Composing Reflections Across Intersecting Lines

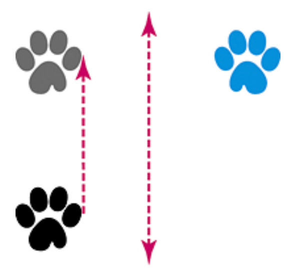
Lines ℓ and m intersect at point C and form an acute angle that measures 70°. What is the image of R reflected first across line ℓ and then across line m? What are the center of rotation and the angle of rotation for the resulting rotation?



Theorem Fundamental Theorem of Isometries

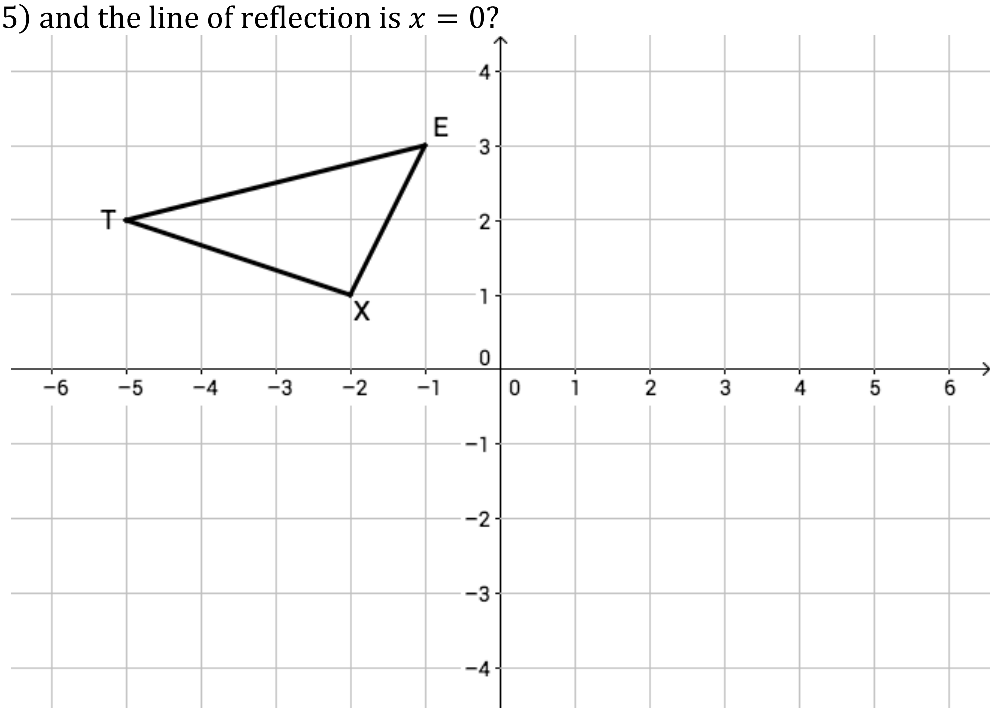
In a plane, one of two congruent figures can be mapped to the other by a composition of at most ________ reflections.

A ______________________________________ is a composition of a translation (a glide) and a reflection across a line parallel to the direction of the translation.



Example Finding a Glide Reflection Image

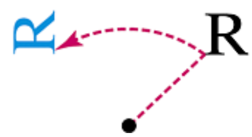
Coordinate Geometry What is the image of ΔTEX for a glide reflection where the translation is $(x, y) \rightarrow (x, y - 5)$ and the line of reflection is $x = 0$?



Theorem Isometry Classification Theorem
There are only four isometries:

Orientations are the same.

____



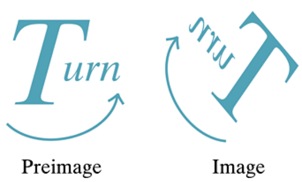
____

Orientations are opposite.

____

____

Example Classifying Isometries
Each transformation is an isometry. Are the orientations of the preimage and image the same or opposite? What type of isometry maps the preimage to the image?

a. 

b.