MAT 222, Final Exam, Summer 2017   Name _____KEY_____

Instructions: You may use a protractor, compass, ruler and calculator for this exam. You may also use three 3x5 index cards, which you will turn in with the exam along with any scrap paper provided by the testing center. It's important to show all work, and explain your reasoning. It is helpful to put a box or circle around your final answer after calculations. Give exact answers unless specifically asked to round.

1. Determine if each statement is True or False. If you mark false, explain why the statement is false, or rewrite the statement as a true one. In the figure, assume lines $\overleftrightarrow{AG}$ and $\overrightarrow{DI}$ are parallel. (2 points each)

   a.   T   (F)   Dilations are a type of isometry.

   b.   (T)   F   The image to the right represents a reflection.

   c.   (T)   F   All objects have at least one rotational symmetry.

   d.   (T)   F   The numbers 10, 24, 26 form a Pythagorean triple.

   e.   T   (F)   A regular hexagon cannot be used alone to tile a plane.

   f.   (T)   F   If all sides of a volume are scaled by a factor of 2, then the measure of the volume is also scaled by a factor of 8.

   g.   T   (F)   A polygon with 10 faces and 14 vertices has 20 edges.

   h.   (T)   F   Angles $\angle CHF$ and angle $\angle AGE$ are supplementary.

   i.   T   (F) $\angle IGB$ and $\angle AGE$ are vertical angles.

   j.   (T)   F $p \rightarrow q$ is logically equivalent to $\sim q \rightarrow \sim p$.

   k.   (T)   F   All parallelograms are trapezoids.

   trapezoids have (at least) one pair of parallel sides

   l.   (T)   F   All right triangles have two acute angles. *(at least)*
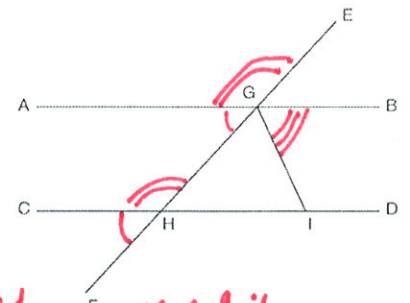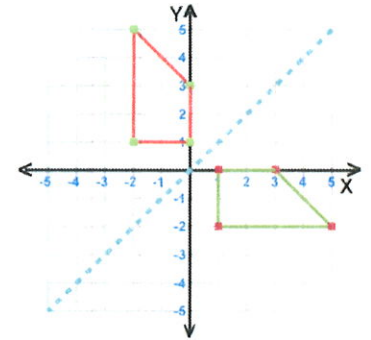
   m.   (T)   F   If two triangles have the same three angles then the triangles are similar.
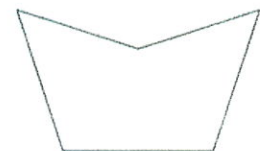
   n.   T   (F)   The centroid is the center of the inscribed circle around a triangle.

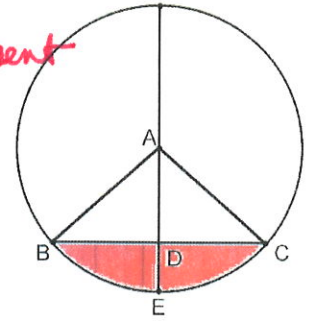   o.   (T)   F   The image to the right is a concave polygon.

p.   T   **F**   The shaded area of the circle graph is a sector.

it's a segment
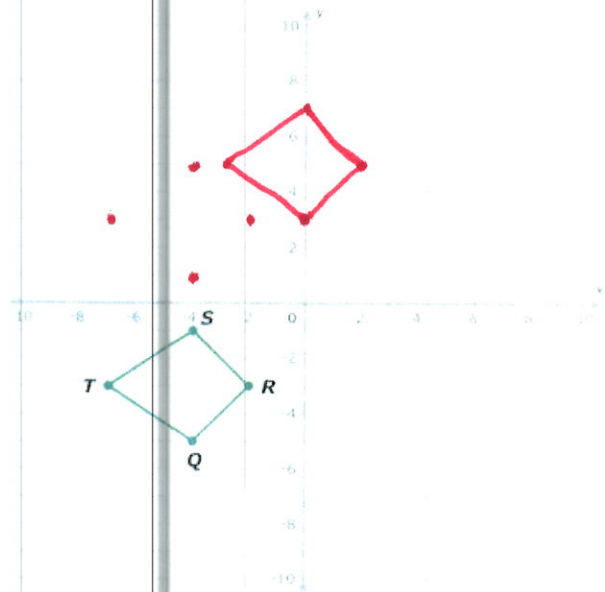
q.   **T**   F   The line segment $\overline{BC}$ is a chord.

2. Use the graphs below to plot the indicated transformations.
    a. Rotation about the origin by 90° clockwise. (5 points)
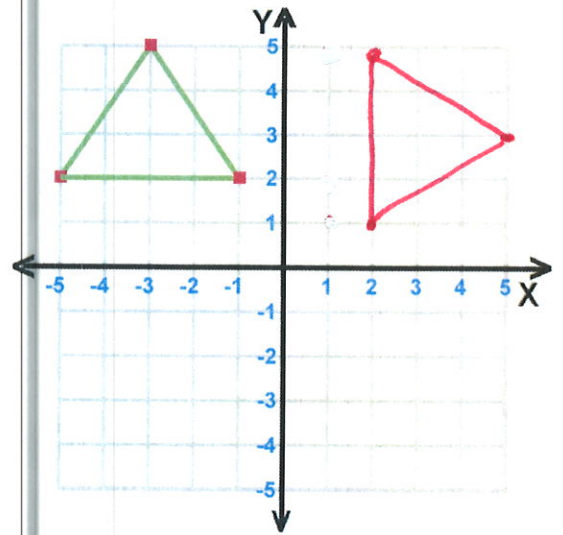
3. Construct a glide reflection of the image below with reflection across the x-axis and translation of $(x, y) \rightarrow (x + 4, y + 2)$. (8 points)

4. Construct a dilation of the object below centered at point P. Determine the scale factor of the dilation in your drawing. (10 points)

mine is
$$\frac{7}{4} = 1.75$$

A

B   C

P
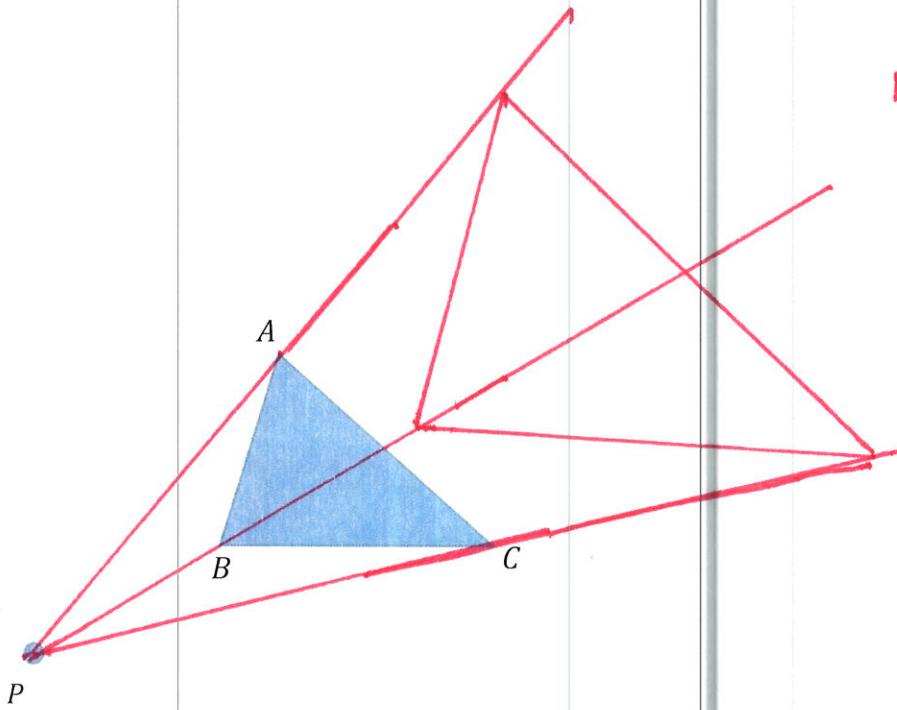
5. Determine the number of lines of symmetry and degree of rotational symmetry for each of the figures below. (18 points)

Lines of Symmetry: __5__	Lines of Symmetry: __0__	Lines of Symmetry: __0__
Rotational Symmetry: __72°__	Rotational Symmetry: __90°__	Rotational Symmetry: __360°__

6. A heptagon has 7 sides.
   a. What is the sum of the measures of the interior angles? (5 points)

   $180(7-2) = 180(5) = 900°$
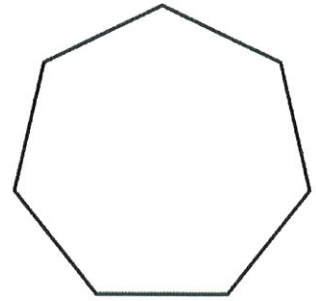
   b. If the heptagon is regular, what is the measure of each interior angle? (5 points)
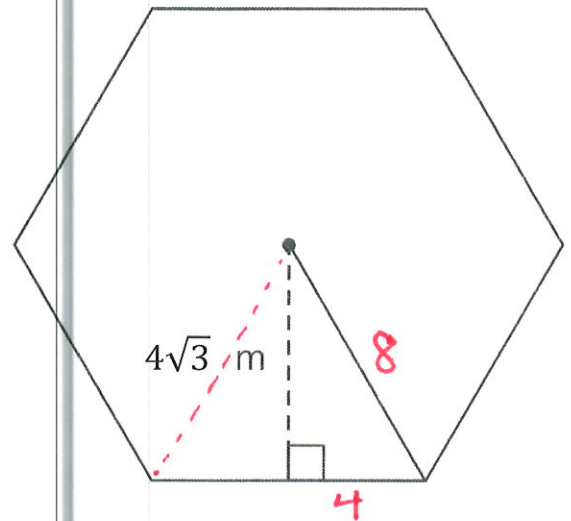
   $\dfrac{900}{7} \approx 128.57°$

   c. What is the measure of each exterior angle? (4 points)

   $\dfrac{360}{7} = 51.43°$

7. Use the image to the right of the regular hexagon to answer the following:
   a. What is the area of the hexagon? (8 points)

   $\dfrac{1}{2}(8)(4\sqrt{3})(6) = 96\sqrt{3} \ m^2$

   $\approx 166.3 \ m^2$

   $4\sqrt{3}$ m   8

   4

   b. What is the perimeter? (5 points)

   $8 \times 6 = 48 \ m$

c. If you scaled the polygon so that the sides were one third the size of the original. What would be the new area? (4 points)

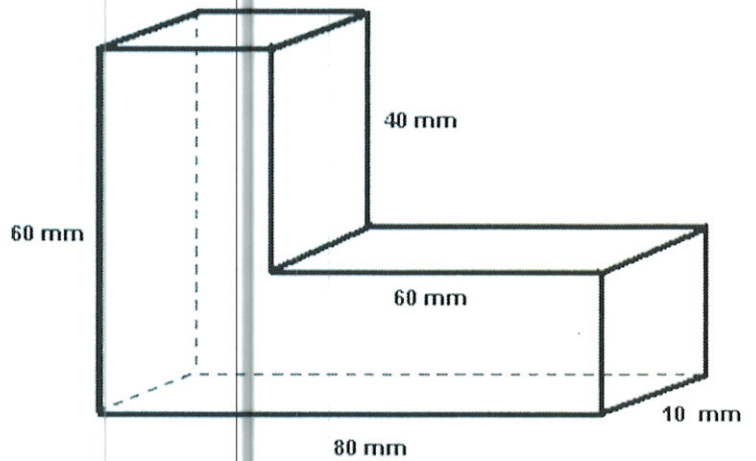$$\frac{96\sqrt{3}}{9} = 32\sqrt{3}\ m^2 \approx 18.48\ m^2$$

d. What would be the new perimeter? (3 points)

$$\frac{48}{3} = 16\ m$$

8. Find the volume of the solid shown to the right. (10 points)

24,000

40 mm

60 mm

60 mm

80 mm

10 mm

9. What is the surface area and volume of a sphere with radius 7 feet? Give your answers in terms of π. (10 points)
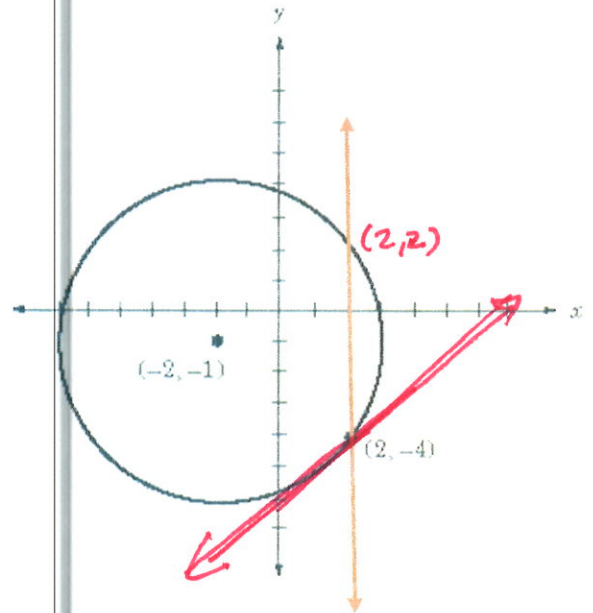
$$SA = 4\pi r^2 = 4\pi(7)^2 = 196\pi\ ft^2$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7)^3 = \frac{1372}{3}\pi\ ft^3$$

10. In the graph below is the circle described by the equation $(x + 2)^2 + (y + 1)^2 = 25$. The line $x = 2$ is shown on the graph.
   a. Find the length of the chord passing through the point $(2, -4)$. Round your answer to the nearest tenth if needed. (5 points)

$x = 2$

$4^2 + (y+1)^2 = 25$
$(y+1)^2 = 9$
$y+1 = \pm 3$
$y = 2, -4$

$2 - (-4) = \boxed{6}$

b. Draw on the graph the tangent line to the circle at the same point. (3 points)
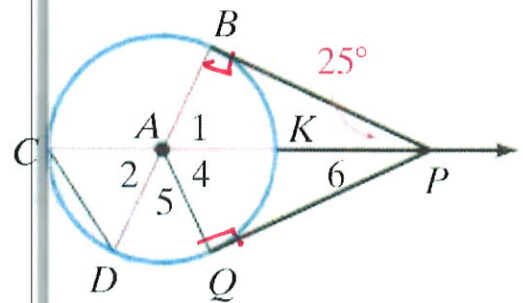
11. Using the diagram to the right, answer the following questions given that $\overline{BD}$ and $\overline{CK}$ are diameters of $\odot A$, and $\overline{BP}$ and $\overline{QP}$ are tangents of $\odot A$.
   a. Find $m\angle 2$ (6 points)

$65°$

b. Find $m\angle 4$ (4 points)

$65°$

12. Suppose that $\vec{w} \parallel \vec{x}$, and that $\vec{y} \parallel \vec{z}$. Use the diagram at the right to state two angles that are: (2 points each)

   a. Alternate interior angles

   $\angle 1, \angle 7$ $\angle 3, \angle 10$

   $\angle 4, \angle 3$

   b. Consecutive/same-side interior angles

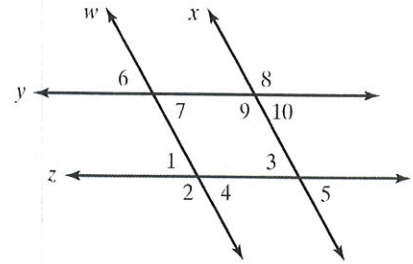 $\angle 7, \angle 9$

   $\angle 9, \angle 3$

   c. Corresponding angles

   $\angle 6, \angle 1$ $\angle 10, \angle 5$ $\angle 4, \angle 5$

   $\angle 7, \angle 4$ $\angle 7, \angle 10$

   d. Alternate exterior angles

   $\angle 4, \angle 6$ $\angle 6, \angle 10$

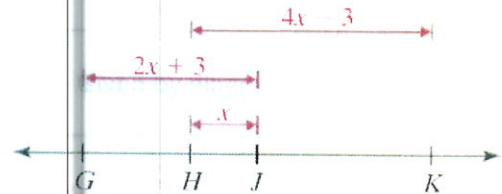 $\angle 1, \angle 5$

13. Using the diagram at the right,

   a. write an algebraic expression for the length of GH, and for the length of JK. (4 points)

   $GH = x + 3$

   $JK = 3x + 3$

   b. If $GK = 50$, find the lengths of each segment GH, HJ, and JK. (4 points)

   $x + 3 + 4x + 3 = 50$ $5x = 44$ $GH = 11.8$

   $5x + 6 = 50$ $x = 8.8$ $HJ = 8.8$

   $JK = 29.4$

14. Write the equation of the line perpendicular to the line $y = \frac{7}{4}x - 3$, passing through the point $(2, -1)$. (6 points)
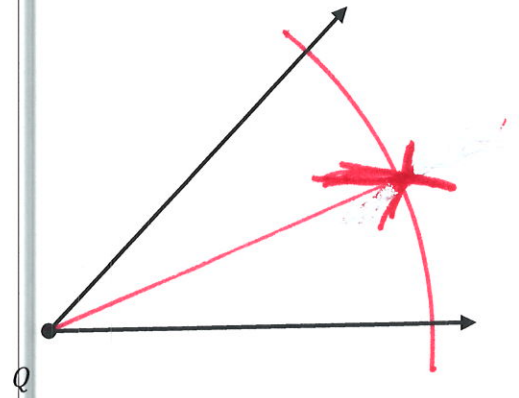
   $m_\perp = -\frac{4}{7}$

   $y + 1 = -\frac{4}{7}(x - 2)$

   $y + 1 = -\frac{4}{7}x + \frac{8}{7}$

   $ -1 \phantom{-\frac{4}{7}x + } -1$

 $y = -\frac{4}{7}x + \frac{1}{7}$

16. Find the angle bisector of the angle shown below. Leave all markings and arcs, and label your points. (8 points)

Q

17. Use your protractor to estimate the measure of $\angle Q$. (4 points)

~47°

18. A supply depot is to be located **equidistant** from three forward bases: A, B, and C. Use your geometry tools (ruler, protractor, compass) to construct the location of the supply depot (call it point P). Leave all of your markings and label point P. (8 points)

Base A

Base C

Base B
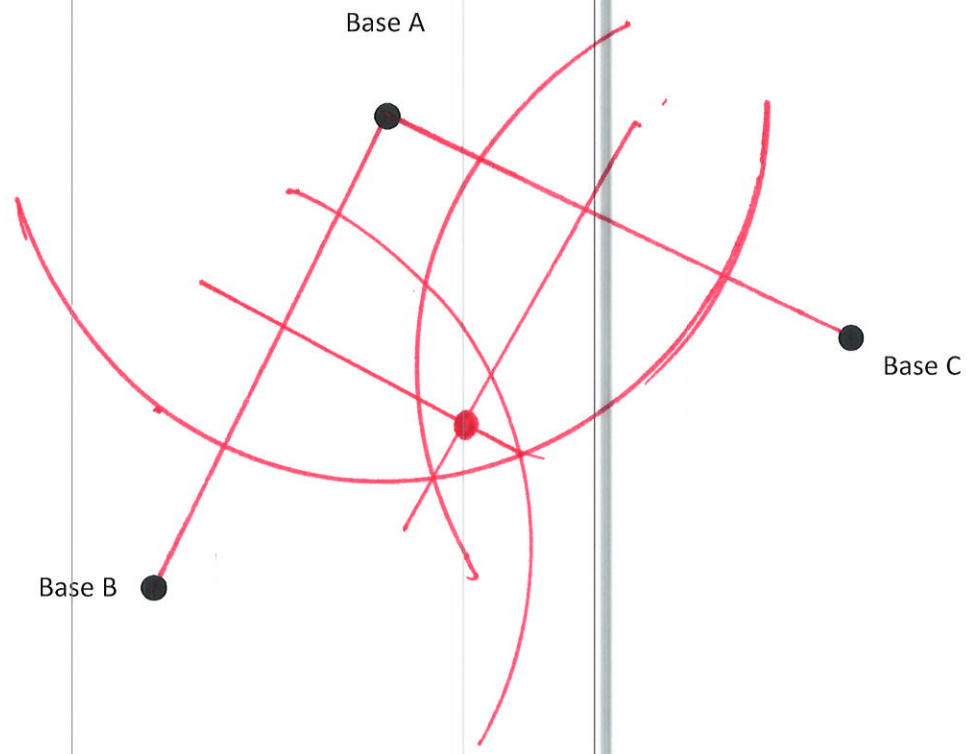
19. The coordinates of a triangle are at
    $A(-2, -3), B(3,4)$, and $C(1,6)$.
    a. Draw the triangle. (3 points)

    b. Find the equation of line of the median
        bisecting $\overline{AB}$. (5 points)

midpoint AB $\left(\dfrac{-2+3}{2}, \dfrac{-3+4}{2}\right) = \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$

Slope $\dfrac{6 - \frac{1}{2}}{1 - \frac{1}{2}} = \dfrac{1\frac{1}{2}}{\frac{1}{2}} = 11$

$Y - 6 = 11(x - 1) =$
$Y - 6 = 11x - 11 \implies y = 11x - 5$

20. Find the height of the tree. (8 points)

$\dfrac{24}{3} = \dfrac{x}{5.5}$

$8 \cdot 5.5 = x$

$x = 44 \; ft.$