MAT 202, Quiz #1, Summer 2017 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Write the systems of equations $\begin{cases} -3x - 2y + 2z = -2 \\ -x - 3y + z = -3 \\ x - 2y + z = -2 \end{cases}$ as an augmented matrix and solve the system by row-reducing. Write the solution as $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

$\left[\begin{array}{ccc|c} -3 & -2 & 2 & -2 \\ -1 & -3 & 1 & -3 \\ 1 & -2 & 1 & -2 \end{array}\right]$ $R_1 \leftrightarrow R_3$, $R_2 + R_3 \to R_2$, $3R_3 + R_1 \to R_3$ $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & -5 & 2 & -5 \\ 0 & -8 & 5 & -8 \end{array}\right]$ $-\frac{1}{5}R_2 \to R_2$, $-\frac{1}{8}R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 1 & -\frac{2}{5} & 1 \\ 0 & 1 & -\frac{7}{8} & 1 \end{array}\right]$ $R_2 - R_3 \to R_3$ $\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 1 & -\frac{2}{5} & 1 \\ 0 & 0 & \frac{9}{40} & 0 \end{array}\right]$ $\frac{40}{9}R_3 \to R_3$

$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -2 \\ 0 & 1 & -\frac{2}{5} & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$z = 0 \quad y = 1$

$x - 2(1) + 0 = -2$

$x = 0$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

2. Solve the system of equations $\begin{cases} x_1 - 2x_2 - 2x_3 - x_4 = -3 \\ -2x_1 + x_2 + x_3 - 2x_4 = -3 \end{cases}$ and write the dependent solution in parametric form.

$\left[\begin{array}{cccc|c} 1 & -2 & -2 & -1 & -3 \\ -2 & 1 & 1 & -2 & -3 \end{array}\right]$ $2R_1 + R_2 \to R_2$ $\left[\begin{array}{cccc|c} 1 & -2 & -2 & -1 & -3 \\ 0 & -3 & -3 & -4 & -9 \end{array}\right]$

$-\frac{1}{3}R_2 \to R_2$ $\left[\begin{array}{cccc|c} 1 & -2 & -2 & -1 & -3 \\ 0 & 1 & 1 & \frac{4}{3} & 3 \end{array}\right]$ $2R_2 + R_1 \to R_1$

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{5}{3} & 3 \\ 0 & 1 & 1 & \frac{4}{3} & 3 \end{array}\right]$

$x_1 + \frac{5}{3}x_4 = 3 \Rightarrow x_1 = -\frac{5}{3}x_4 + 3$

$x_2 + x_3 + \frac{4}{3}x_4 = 3 \Rightarrow x_2 = -x_3 - \frac{4}{3}x_4 + 3$

$x_3 = x_3$

$x_4 = x_4$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -\frac{5}{3} \\ -\frac{4}{3} \\ 0 \\ 1 \end{bmatrix} x_4 + \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -5 \\ -4 \\ 0 \\ 3 \end{bmatrix} s + \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

3. Determine if the matrices are in reduced row echelon form, row echelon form, or neither.

a. $\begin{bmatrix} 1 & 0 & -3 & 4 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

pivots

reduced echelon form

b. $\begin{bmatrix} 1 & 0 & 0 & 4 & -1 \\ 0 & 0 & 1 & 5 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}$

neither

could be in rref if $R_2 \leftrightarrow R_3$