

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. What is the dimension of the vector spaces described below (as a vector space over \mathbb{R})?

a. \mathbb{R}^4 **4**

c. P_5 **6**

b. \mathbb{C}^2 **4**

d. $M_{6 \times 9}$ **54**

e. The space spanned by $\{x + 1, x^3 + x^2, x - x^2\}$.

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}^T \quad \mathbf{3}$$

2. Consider the matrix $A = \begin{bmatrix} -1 & 1 & 1 & 0 & -2 \\ 1 & -3 & -2 & -2 & 5 \\ 1 & -1 & -1 & 0 & 2 \\ -1 & 2 & 3 & 1 & -5 \end{bmatrix}$. Find:

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. A basis for $\text{Col } A$

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

b. A basis for $\text{Nul } A$

$$\begin{aligned} x_1 &= -x_4 \\ x_2 &= -x_4 + x_5 \\ x_3 &= x_5 \\ x_4 &= x_4 \\ x_5 &= x_5 \end{aligned}$$

$$\vec{x} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} x_5$$

c. Rank A

3

3. If $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \right\}$, find $\vec{x} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$ in this basis.

$$P_B [\vec{x}]_B = \vec{x}$$

$$[\vec{x}]_B = P_B^{-1} \vec{x} = \begin{bmatrix} 1/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & 1/3 \\ -1/3 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$$