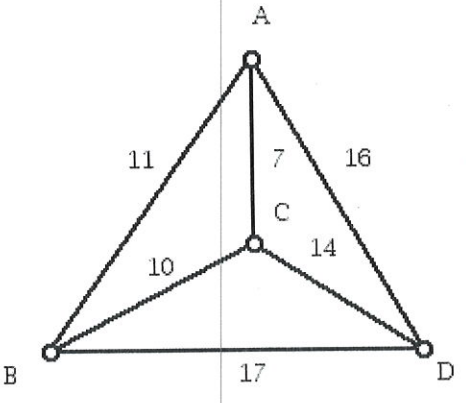
MAT 100, Exam #2, Summer 2017 Name KEY

**Instructions**: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Use the Brute Force algorithm to find the minimal cost Hamilton circuit for the $K_4$ graph shown. State the final cost of the minimal circuit. (10 points)



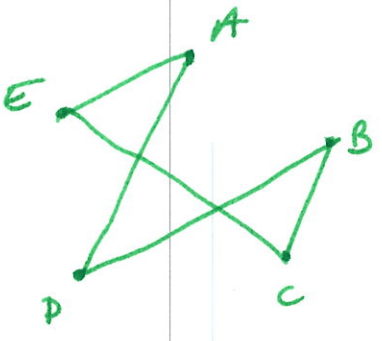

ABCDA = 11 + 10 + 14 + 16 = 51

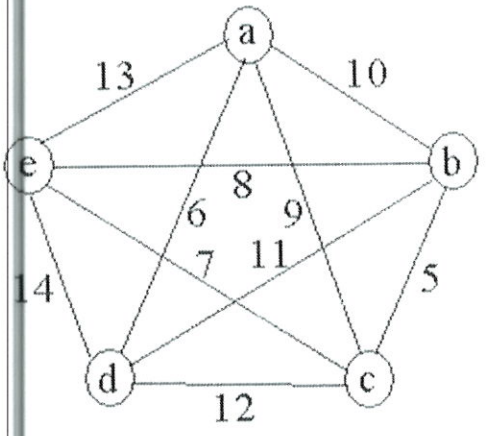
ACBDA = 7 + 10 + 17 + 16 = 50

ACDBA = 7 + 14 + 17 + 11 = 49

2. For the complete graph below, find the approximate minimal cost Hamilton circuit using the indicated method. State the final cost of the circuit. (6 points each)

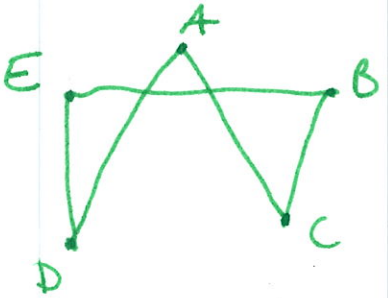
a. Nearest Neighbor starting at A



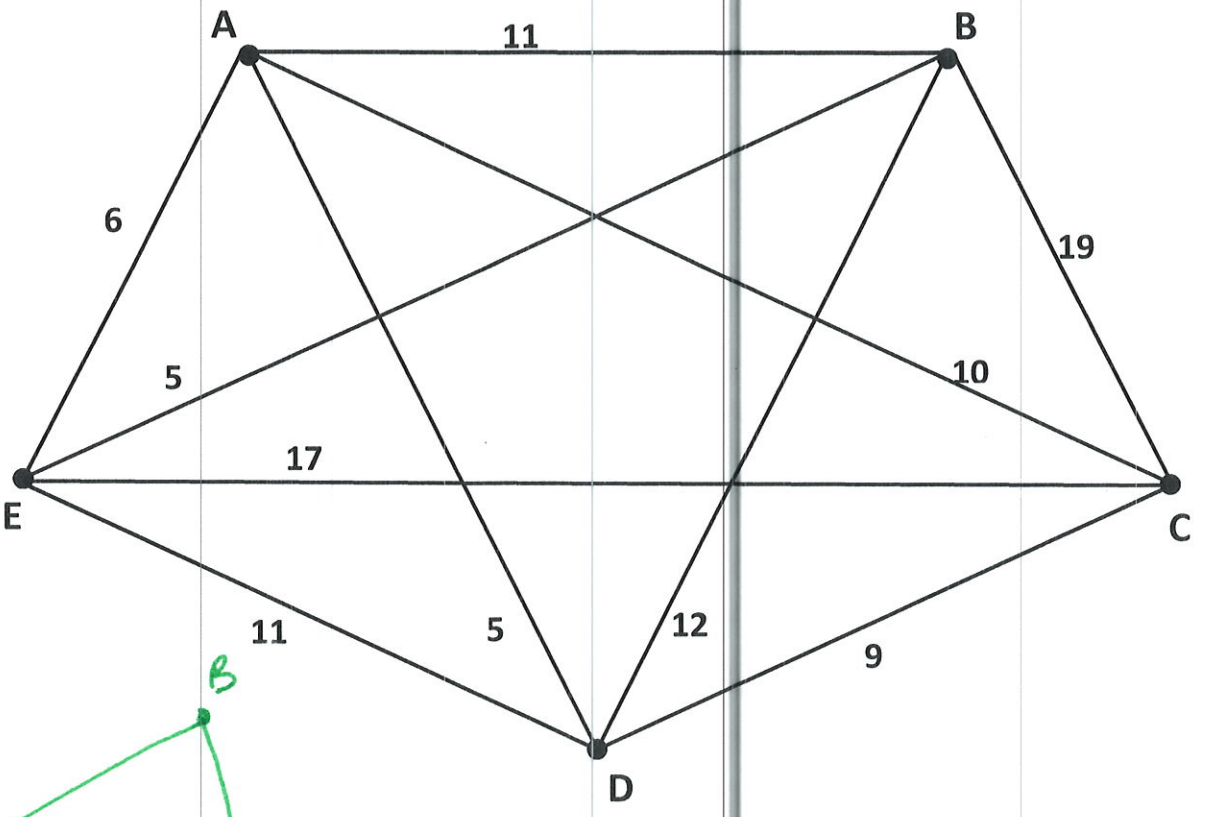

6 + 11 + 5 + 7 + 13 = 42

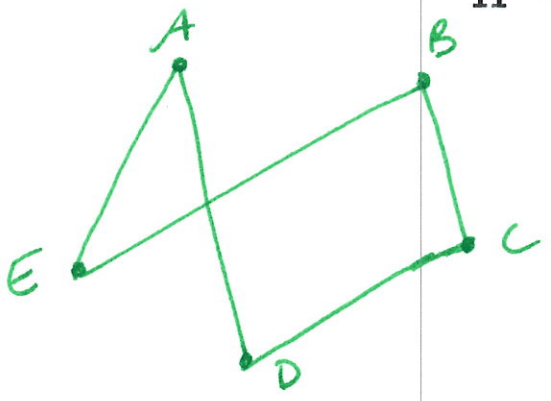
b. Nearest Neighbor starting at D




6 + 9 + 5 + 8 + 14 = 42

3. Use the Cheapest Link/Sorted Edges algorithm on the graph below to find the approximate minimal cost Hamilton Circuit. Clearly state the cost of the final circuit. (8 points)

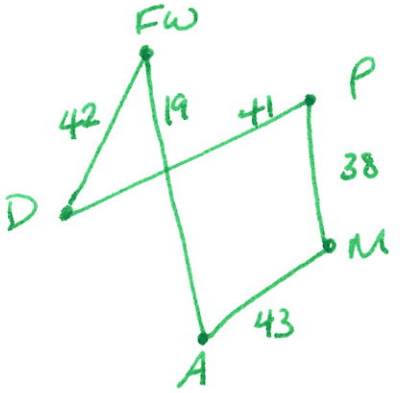



5+5+6+9+19=44

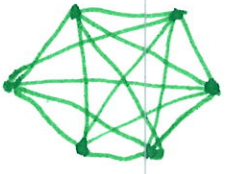
4. Use the mileage table shown below and the Nearest Neighbor algorithm starting from Plano to find the minimal cost Hamilton circuit for traveling between the cities in the table. Be sure to clearly state the final cost of the circuit. (8 points)

| | Plano | Mesquite | Arlington | Denton |
|---|---|---|---|---|
| Fort Worth | 54 | 52 | ~~19~~ | ~~42~~ |
| Plano | | ~~38~~ | 53 | 41 |
| Mesquite | | | ~~43~~ | 56 |
| Arlington | | | | 50 |




42+19+43+38+41 = 183

5. Draw a $K_6$ graph. (5 points)



6. Apportion the following table of an imaginary state legislature according to Hamilton's Method if there are 183 seats to be apportioned. (10 points)

| STATE | POPULATION | STANDARD QUOTA | LOWER QUOTA | UPPER QUOTA | +1 | FINAL APPORTIONMENT |
|---|---|---|---|---|---|---|
| TOGO | 89,290 | 49.48 | 49 | 50 | +1 | 50 |
| IZUMI | 97,974 | 54.29 | 54 | 55 | | 54 |
| SAKATA | 59,597 | 33.02 | 33 | 34 | | 33 |
| OGURA | 9,782 | 5.42 | 5 | 6 | | 5 |
| TOYAMA | 25,742 | 14.26 | 14 | 15 | | 14 |
| KOISO | 47,869 | 26.53 | 26 | 27 | +1 | 27 |
| TOTALS | 330,254 | | 181 | | | 183 |

SD = 1804.67

7. Apportion the following table of an imaginary state legislature according to Adams' Method if there are 183 seats to be apportioned. Suggested Modified Divisors are 1780, 1800, 1840. (10 points)

| STATE | POPULATION | STANDARD QUOTA | MODIFIED QUOTA | MODIFIED QUOTA | MODIFIED QUOTA | FINAL APPORTIONMENT |
|---|---|---|---|---|---|---|
| TOGO | 89,290 | 49.48 | 48.52 | | | 49 |
| IZUMI | 97,974 | 54.29 | 53.25 | | | 54 |
| SAKATA | 59,597 | 33.02 | 32.39 | | | 33 |
| OGURA | 9,782 | 5.42 | 5.32 | | | 6 |
| TOYAMA | 25,742 | 14.26 | 13.99 | | | 14 |
| KOISO | 47,869 | 26.53 | 26.02 | | | 27 |
| TOTALS | 330,254 | | | | | 183 |

SD = 1804.67

8. Apportion the following table of an imaginary state legislature according to Huntington-Hill's Method if there are 183 seats to be apportioned. (10 points)

| STATE | POPULATION | STANDARD QUOTA | LOWER QUOTA | UPPER QUOTA | $\sqrt{LQ * UQ}$ | FINAL APPORTIONMENT |
|---|---|---|---|---|---|---|
| TOGO | 89,290 | ~~48.48~~ 49.61 | ~~49~~ | ~~50~~ | 49.497 | 50 |
| IZUMI | 97,974 | ~~54.24~~ 54.43 | 54 | 55 | 54.4977 | ~~54~~ |
| SAKATA | 59,597 | ~~33.02~~ 33.11 | 33 | 34 | 33.496 | 33 |
| OGURA | 9,782 | ~~5.42~~ 5.43 | 5 | 6 | 5.477 | 5 |
| TOYAMA | 25,742 | ~~14.26~~ 14.30 | 14 | 15 | 14.491 | 14 |
| KOISO | 47,869 | ~~26.53~~ 26.59 | 26 | 27 | 26.495 | 27 |
| TOTALS | 330,254 | | | | | 183 |

SD = 1804.67 MQ = 1800

9. The following examples represent apportionment paradoxes (or quota rule violations). Determine which one is represented by each example and justify your reasoning. (5 points each)
   a. The following two different apportionments of the same population.

| School | Original Apportionment | New Apportionment |
|---|---|---|
| Cascades | 9 | 9 |
| Seven Oak | 11 | 10 |
| Riverview | 6 | 7 |
| Pioneer | 4 | 5 |
| Hamilton Creek | 5 | 6 |
| | =35 seats | =37 seats |

Alabama paradox
seats changed
lost a seat

   b. The following table shows apportionments for successive censuses with 50 seats each.

| State | Population #1 | Apportionment #1 | Population #2 | Apportionment #2 |
|---|---|---|---|---|
| Jefferson | 60,000 | 8 | 60,000 | 8 |
| Clay | 31,200 | 4 | 31,200 | 5 |
| Madison | 69,200 | 10 | 72,400 | 10 |
| Jackson | 81,600 | 11 | 81,600 | 11 |
| Franklin | 118,000 | 17 | 118,400 | 16 |
| Totals | 360,000 | 50 | 363,600 | 50 |

Clay 0% increase

Franklin $\frac{400}{118000} = .0033898 > 0$

Population

Franklin grew but lost a seat vs. Clay stayed same but gained

c. The following table represents the apportionment of 6 states using Jefferson's Method.

| State | Population | Standard Quota (SD = 50,000) | Lower Quota | Modified Quota (D = 49,500) | Jefferson apportionment |
|---|---|---|---|---|---|
| A | 1 646 000 | 32 92 | 32 | 33 25 | 33 |
| B | 6 936 000 | **138.72** | 138 | 140 12 | **140** |
| C | 154 000 | 3 08 | 3 | 3 11 | 3 |
| D | 2 091 000 | 41 82 | 41 | 42 24 | 42 |
| E | 685 000 | 13 70 | 13 | 13 84 | 13 |
| F | 988 000 | 19 76 | 19 | 19 96 | 19 |
| Total | 12 500 000 | 250 00 | 246 | | 250 |

Quota rule violations 140 is above upper quota of 139.

10. Describe (Balinksi-)Young's Impossibility Theorem and why it's important for understanding apportionment. (5 points)

it says that no apportionment method is perfectly fair. All methods will either have paradoxes or sometimes produce quota rule violations.

11. Suppose that $A$ is the set of letters in the name ELIZABETH and $B$ is the set of letters in the name PENELOPE. (3 points each)
    a. List the elements of set $A$

    $\{E, L, I, Z, A, B, T, H\}$ (don't list E twice)

    b. List the elements of set $B$

    $\{P, E, N, L, O\}$ (only list P, E once each)

    c. Find $A \cup B$

    $\{E, L, I, Z, A, B, T, H, P, N, O\}$

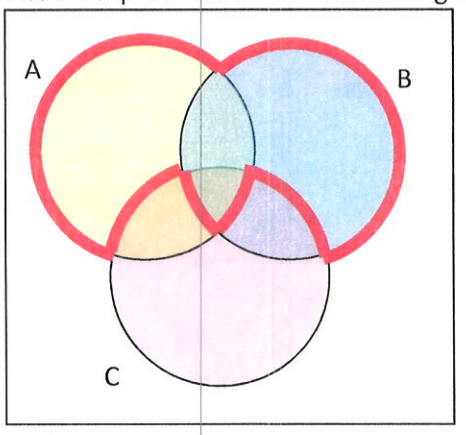
    d. Find $A \cap B$

    $\{E, L\}$

    e. If $U$ is the set of all letters in the standard English alphabet, what is $A^c$?

    $\{C, D, F, G, J, K, M, N, O, P, Q, R, S, U, V, W, X, Y\}$

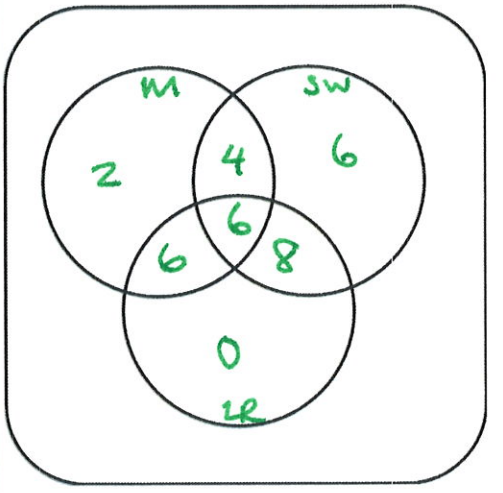
12. Write an expression for the shaded region. (6 points)




$(A-C) \cup (B-C) \cup (A \cap B)$

13. Use the information below to fill in the Venn Diagram, then determine: (5 points each + diagram)
    a. How many students have seen exactly one of these movies? 8
    b. How many had seen only *Star Wars*? 6

18 had seen *The Matrix* (*M*)
24 had seen *Star Wars (SW)*
20 had seen *Lord of the Rings (LotR)*
10 had seen *M* and *SW*
14 had seen *LotR* and *SW*
12 had seen *M* and *LotR*
6 had seen all three




14. The 100 tiles in Scrabble are distributed as follows:

| Tile | Number | Tile | Number | Tile | Number | Tile | Number |
|---|---|---|---|---|---|---|---|
| Blank | 2 | A | 9 | B | 2 | C | 2 |
| D | 4 | E | 12 | F | 2 | G | 3 |
| H | 2 | I | 9 | J | 1 | K | 1 |
| L | 4 | M | 2 | N | 6 | O | 8 |
| P | 2 | Q | 1 | R | 6 | S | 4 |
| T | 6 | U | 4 | V | 2 | W | 2 |
| X | 1 | Y | 2 | Z | 1 | | |

a. What is the probability of selecting an N as the first tile in a game? (3 points)

$\frac{6}{100} = \frac{3}{50} = 6\%$

b. What is the probability of selecting a vowel (not Y or blanks) as the first tile? (3 points)

$$\frac{9+12+9+8+4}{100} = \frac{42}{100} = \frac{21}{50} = 42\%$$

c. What is the probability of not selecting a vowel? (3 points)

$$1 - 42\% = 58\%$$

d. What is the probability of selecting the word BOX in order from the first three tiles in a game? (5 points)

$$\frac{2}{100} * \frac{8}{99} * \frac{1}{98} = 1.649 \times 10^{-5}$$

15. Use the following table to calculate the probabilities requested. (4 points each)

| | | STUDENT'S CHOICE | | Total |
|---|---|---|---|---|
| | | Art degree | Science degree | |
| GROUP | Boys | 25 | 50 | 75 |
| | Girls | 55 | 20 | 75 |
| | Total | 80 | 70 | 150 |

a. What is the probability of a randomly selected person from this study is a girl?

$$\frac{75}{150} = \frac{1}{2} = 50\%$$

b. What is the probability of a randomly selected person from this study being a girl and wants a science degree?

$$\frac{20}{150} = \frac{2}{15} = 13.\overline{3}\%$$

c. What is the probability of a randomly selected person from this study being a girl or wanting a science degree?

$$\frac{75}{150} + \frac{70}{150} - \frac{20}{150} = \frac{125}{150} = 83.\overline{3}\%$$

d. What is the probability of being a girl given that the person wants a science degree?

$$\frac{20}{70} = \frac{2}{7} = 28.57\%$$

e. Are the variables gender and degree choice independent? Why or why not? Show calculations to justify your answer.

they are not independent
the prob. of being a girl is 50%, but given a science degree, its only 28.57%. They are not the same

16. Determine the **number** of outcomes in each of the following scenarios. (5 points each)

a. A local area network requires eight characters for a username and is case sensitive, but the character can use numbers only in the last three digits (i.e. capital and lower case letters in the first five digits, but letters and numbers in the last three). How many usernames of this type are there?

$$\overline{52} \cdot \overline{52} \cdot \overline{52} \cdot \overline{52} \cdot \overline{52} \cdot \overline{62} \cdot \overline{62} \cdot \overline{62} = 9 \times 10^{13}$$

b. Suppose that a lottery has 44 balls, and someone needs 6 matches in any order to win the top prize. How many possible winning number combinations are possible?

$$44C6 = \binom{44}{6} = 7{,}059{,}052$$

c. Suppose that ten horses are in a particular race. How many ways can the top four horses finish?

$$10P4 = 5040$$

17. Find the probability of each of the following scenarios. (5 points each)
    a. Social Security numbers are composed of 9 digits, and each digit can be any number from 0-9. What is the probability that a randomly selected Social Security number will be all odd numbers?

$$\frac{5^9}{10^9} = .00195\ldots$$

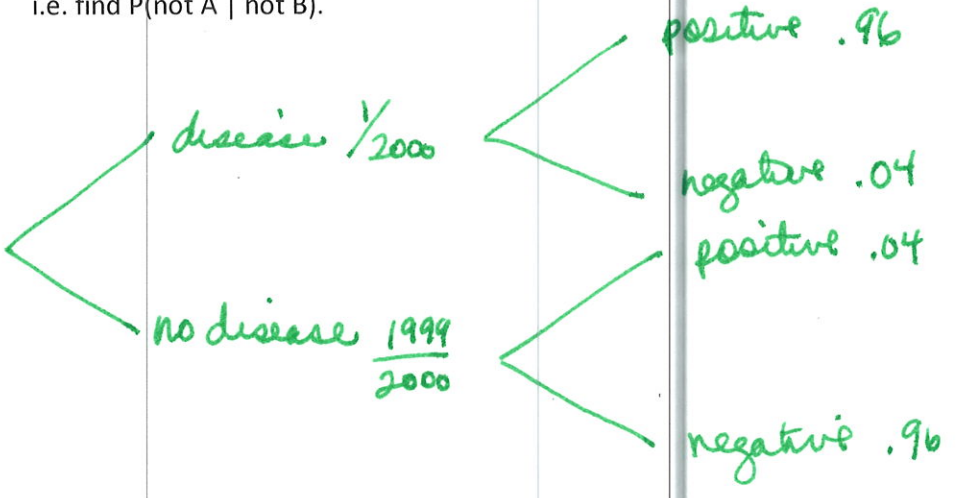
    b. What is the probability of getting a pair of aces and a pair of kings in a 5-card poker hand?

$$\frac{(4C2)(4C2)\cdot 44}{52C5} = 6.09 \times 10^{-4}$$

18. Evaluate the following expressions. (3 points each)

| a. $\binom{9}{4}$ | b. ${}_3^7P$ | c. ${}_5^{12}C$ | d. 0! |
|---|---|---|---|
| 126 | 210 | 792 | 1 |

19. A certain virus infects one in every 2000 people. A test used to detect the virus in a person is positive 96% of the time if the person has the virus and 4% of the time if the person does not have the virus. Let A be the event "the person is infected" and B be the event "the person tests positive". [Hint: use a tree diagram.] (5 points each)
    a. Find the probability that a person has the virus given that they have tested positive, i.e. find P(A | B).
    b. Find the probability that a person does not have the virus given that they test negative, i.e. find P(not A | not B).




$$\frac{\frac{1}{2000}(.96)}{\frac{1}{2000}\cdot(.96)+\frac{1999}{2000}(.04)} = .01186$$

a 1.19%

$$\frac{\frac{1999}{2000}(.96)}{\frac{1999}{2000}(.96)+\frac{1}{2000}(.04)} = .999979$$

b

20. Suppose that the odds against an event are 7:9. What is the probability of the event? (5 points)

$7+9=16$

$\boxed{\frac{9}{16}}$

21. Suppose that the probability of an event is $P(A) = \frac{16}{33}$. What are the odds for the event? (5 points)

$33-16=17$

$\boxed{16:17}$