MAT 212, Quiz #5, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Determine the region in the plane where a solution is guaranteed to exist for $y' = \frac{t+y}{(t+1)(t^2+y^2-4)}$. Sketch a graph of the region.

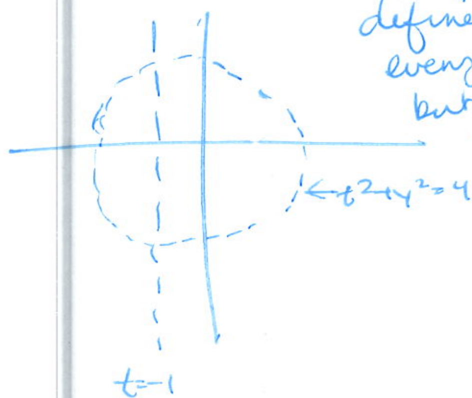
$f(t,y) = \frac{t+y}{(t+1)(t^2+y^2-4)}$

$t+1 \neq 0 \Rightarrow t \neq -1$

$t^2+y^2-4 \neq 0$

$t^2+y^2 \neq 4$

Circle of radius 2



defined everywhere but dotted paths.

$\frac{\partial f}{\partial y} = \frac{1(t+1)(t^2+y^2-4) - (t+1)(2y)}{(t+1)^2(t^2+y^2-4)^2}$ no new DNE points

2. Verify the ODE $(x + \arctan y)dx + \left(\frac{x+y}{1+y^2}\right) dy = 0$ is exact. Solve the equation.

M N

$\frac{\partial M}{\partial y} = \frac{1}{y^2+1}$

$\frac{\partial N}{\partial x} = \frac{1}{y^2+1}$

yes, it is exact

$\int x + \arctan y \, dx = \frac{1}{2}x^2 + y\arctan y + f(y)$

$\int \frac{x}{1+y^2} + \frac{y}{1+y^2} \, dy = x\arctan y + \frac{1}{2}\ln|1+y^2| + g(x)$

$$\boxed{\varphi:\ x\arctan y + \frac{1}{2}x^2 + \frac{1}{2}\ln|1+y^2| = C}$$