MAT 212, Quiz #4, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the separable ODE $\frac{dy}{dx} = 2x \sec y$

$\frac{dy}{dx} = 2x \sec y \Rightarrow \frac{dy}{\sec y} = 2x\,dx =$

$\int \cos y\,dy = \int 2x\,dx \Rightarrow \boxed{\sin y = x^2 + C}$

or $\arcsin(x^2 + C) = y$

2. Solve the linear IVP $y' + 2xy = x, y(0) = -2$ using the method of integrating factors. [Hint: $\mu = e^{\int p(t)dt}$.]

$p(x) = 2x$

$\mu = e^{\int 2x\,dx} = e^{x^2}$

$e^{x^2}y' + 2xe^{x^2}y = xe^{x^2}$

product rule

$\int (e^{x^2}y)' = \int xe^{x^2}\,dx$

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2}e^u\,du$
$= \frac{1}{2}e^u + C$

$e^{x^2}y = \frac{1}{2}e^{x^2} + C$

$\frac{e^{x^2}y}{e^{x^2}} = \frac{\frac{1}{2}e^{x^2} + C}{e^{x^2}}$

$y = \frac{1}{2} + Ce^{-x^2}$

$x = 0$

$-2 = \frac{1}{2} + Ce^0$

$-2 = \frac{1}{2} + C$

$-\frac{5}{2} = C$

$y = \frac{1}{2} - \frac{5}{2}e^{-x^2}$