

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use the modified Euler method to estimate the solution to $y' = \frac{2x^3}{y}$, $y(1) = 3$ at $t = 2$ in three steps. $y_{n+1} = y_n + hf \left[t_n + \frac{1}{2}h, y_n + \frac{1}{2}hf(t_n, y_n) \right]$

$\frac{2-1}{3} = \frac{1}{3} = \Delta t = h$

n	t_n	y_n	$t_n + \frac{1}{2}h$	$f(t_n, y_n)$	$y_n + \frac{1}{2}hf_n$	$f(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hf_n)$	y_{n+1}
0	1	3	$\frac{7}{6}$	$\frac{2(1)^3}{3} = \frac{2}{3}$	$3 + \frac{1}{6}(\frac{2}{3}) = \frac{28}{9}$	$\frac{2(\frac{7}{6})^3}{\frac{28}{9}} = \frac{49}{48}$	$3 + \frac{1}{3}(\frac{49}{48}) = \frac{481}{144}$
1	$\frac{4}{3}$	$\frac{481}{144}$	$\frac{3}{2}$	$\frac{2(\frac{4}{3})^3}{\frac{48}{144}} = \frac{2048}{1443}$	$\frac{481}{144} + \frac{1}{6}(\frac{2048}{1443}) \approx 3.5768$	$\frac{2(\frac{3}{2})^3}{3.5768} \approx 1.887$	$\frac{481}{144} + \frac{1}{3}(1.887) \approx 3.96933...$
2	$\frac{5}{3}$	3.96933	$\frac{11}{6}$	$\frac{2(\frac{5}{3})^3}{3.96933} \approx 2.3327...$	4.358...	$\frac{2(\frac{11}{6})^3}{4.358...} \approx 1.54246...$	$3.96933 + \frac{1}{3}(1.54246...) \approx 4.48348...$
3	2	4.48...					

$y(2) \approx 4.48348...$