MAT 212, Quiz #21, Summer 2016 Name KEY

**Instructions**: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite the second order ODE $2y'' + 7y' + 13y = e^{-t}$ as a system of first order equations. (You don't need to solve it.)

$y = x_1$
$y' = x_1' = x_2$
$y'' = x_2'$

$2x_2' + 7x_2 + 13x_1 = e^{-t}$

$x_1' = x_2$
$x_2' = -\frac{13}{2}x_1 - \frac{7}{2}x_2 + \frac{1}{2}e^{-t}$

$\vec{x}' = \begin{bmatrix} 0 & 1 \\ -13/2 & -7/2 \end{bmatrix}\vec{x} + \begin{bmatrix} 0 \\ 1/2 e^{-t} \end{bmatrix}$

2. Find the general solution to the system $\vec{x}' = \begin{bmatrix} 1 & -5 \\ 1 & -1 \end{bmatrix}\vec{x}$.

$(1-\lambda)(-1-\lambda)+5$
$\lambda^2 - 1 + 5 = 0$
$\lambda^2 + 4 = 0$
$\lambda = \pm 2i$

$\begin{bmatrix} 1-2i & -5 \\ 1 & -1-2i \end{bmatrix}$

$x_1 = (1+2i)x_2$
$x_2 = x_2$

$\vec{v}_1 = \begin{bmatrix} 1+2i \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1+2i \\ 1 \end{bmatrix} e^{2it} = \begin{bmatrix} 1+2i \\ 1 \end{bmatrix}(\cos 2t + i \sin 2t) = \begin{pmatrix} \cos 2t + i\sin 2t + 2i\cos 2t - 2\sin 2t \\ \cos 2t + i\sin 2t \end{pmatrix}$

$\vec{x}(t) = c_1 \begin{pmatrix} \cos 2t - 2\sin 2t \\ \cos 2t \end{pmatrix} + c_2 \begin{pmatrix} \sin 2t + 2\cos 2t \\ \sin 2t \end{pmatrix}$