MAT 212, Quiz #20, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 7 & -5 \\ 4 & 3 \end{bmatrix}$.

$(7-\lambda)(3-\lambda)+20$

$\lambda^2-10\lambda+21+20=0$

$\lambda^2-10\lambda+41=0$

$\lambda = \frac{+10 \pm \sqrt{100-164}}{2}$

$= \frac{10 \pm \sqrt{-64}}{2} = \frac{10 \pm 8i}{2}$

$\lambda_{1,2} = 5 \pm 4i$

$\begin{bmatrix} 7-5-4i & -5 \\ 4 & 3-5-4i \end{bmatrix} = \begin{bmatrix} 2-4i & -5 \\ 4 & -2-4i \end{bmatrix} \div 2$

$2x_1 - (1+2i)x_2 = 0$

$x_1 = \frac{(1+2i)}{2} x_2$

$x_2 = x_2$ $\quad \vec{v}_1 = \begin{pmatrix} 1+2i \\ 2 \end{pmatrix}$

$\$ \quad \vec{v}_2 = \begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$

2. Solve the ODE $y'' + 5y' + 6y = 2\cos 4t, y(0) = 1, y'(0) = 2$ using Laplace transforms.

$s^2Y(s) - s(1) - 2 + 5(sY(s) - 1) + 6Y(s) = \frac{2s}{s^2+16}$

$(s^2+5s+6)Y(s) - s - 2 - 5 = \frac{2s}{s^2+16}$

$Y(s) = \frac{(s+7)(s^2+16)+2s}{(s^2+5s+6)(s^2+16)} = \frac{s^3+7s^2+18s+112}{(s+2)(s+3)(s^2+16)}$

$\frac{A}{s+2} + \frac{B}{s+3} + \frac{Cs+D}{s^2+16} = \frac{(As+3A)(s^2+16)+(Bs+2B)(s^2+16)+(Cs+D)(s^2+5s+6)}{(s+2)(s+3)(s^2+16)}$

$As^3+16As+3As^2+48A+Bs^3+16Bs+2Bs^2+32B+Cs^3+5Cs^2+6Cs+Ds^2+5Ds+6D$

$A+B+C=1$

$3A+2B+5C+D=7$

$16A+16B+6C+5D=18$

$48A+32B+6D=112$

$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 1 \\ 3 & 2 & 5 & 1 & 7 \\ 16 & 16 & 6 & 5 & 18 \\ 48 & 32 & 0 & 6 & 112 \end{array}\right]$

$A = 24/5$

$B = -94/25$

$C = -1/25$

$D = 8/25$

$\frac{24}{5}\left(\frac{1}{s+2}\right) - \frac{94}{25}\left(\frac{1}{s+3}\right) - \frac{1}{25}\left(\frac{s}{s^2+16}\right) + \frac{2}{25}\left(\frac{4}{s^2+16}\right)$

$\frac{24}{5}e^{-2t} - \frac{94}{25}e^{-3t} - \frac{1}{25}\cos 4t + \frac{2}{25}\sin 4t$

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	t^p, $p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n+\frac{1}{2}}$, $n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{\left(s^2+a^2\right)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{\left(s^2+a^2\right)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{\left(s^2+a^2\right)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at}$, $n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			