

**Instructions:** Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Perform the indicated operation.

a. Differentiate  $y = 5e^{-3t} \cos 4t + t \ln(\cos t)$

$$y' = 5(-3)e^{-3t} \cos 4t + 5e^{-3t} (4)(-\sin 4t) + \ln|\cos t| + t \cdot \frac{-\sin t}{\cos t}$$

$$= \boxed{-15e^{-3t} \cos 4t - 20e^{-3t} \sin 4t + \ln|\cos t| - \frac{t \sin t}{\cos t}}$$

b. Integrate  $\int \frac{2t+1}{(t^2+1)(t-3)} dt$

$$\frac{At+B}{t^2+1} + \frac{C}{t-3}$$

$$At^2 + Bt + 3At - 3B + Ct^2 + C = 2t + 1$$

$$A + C = 0$$

$$-3A + B = 2$$

$$-3B + C = 1$$

$$A = -\frac{7}{10}$$

$$B = -\frac{1}{10}$$

$$C = \frac{7}{10}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -3 & 1 & 0 & 2 \\ 0 & -3 & 1 & 1 \end{array} \right]$$

$$= \int -\frac{7}{10} \cdot \frac{t}{t^2+1} - \frac{1}{10} \cdot \frac{1}{t^2+1} + \frac{7}{10} \cdot \frac{1}{t-3} dt$$

$$= \boxed{-\frac{7}{20} \ln|t^2+1| - \frac{1}{10} \arctan t + \frac{7}{10} \ln|t-3| + C}$$

2. Write the following expressions in standard  $(a + bi)$  form.

a.  $e^{(-2+i)t}$

$$e^{-2t} \cdot e^{it}$$

$$e^{-2t} (\cos t + i \sin t)$$

$$e^{-2t} \cos t + i e^{-2t} \sin t$$

b.  $t^{1-i}$

$$t^1 \cdot t^{-i}$$

$$t^1 (e^{\ln t})^{-i} = t \cdot e^{-i(\ln t)}$$

$$t (\cos(\ln t) - i \sin(\ln t))$$

$$= t \cos(\ln t) - i t \sin(\ln t)$$