MAT 212, Quiz #19, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find $\mathcal{L}^{-1}\{F(s)\}$ for

a. $F(s) = \frac{s}{(s+3)(s^2+1)}$

$\frac{1}{s+3} \cdot \frac{s}{s^2+1}$

as a convolution: $\int_0^t e^{-3(t-\tau)} \cos\tau \, d\tau$

$\frac{A}{s+3} + \frac{Bs+C}{s^2+1} = \frac{As^2+A+Bs^2+3Bs+Cs+3C}{(s+3)(s^2+1)}$

$A+B=0$
$3B+C=1$
$A+3C=0$

$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 3 & 0 \end{array}\right]$

$A = -\frac{3}{10}, B = \frac{3}{10}, C = \frac{1}{10}$

$-\frac{3}{10}\left(\frac{1}{s+3}\right) + \frac{3}{10}\left(\frac{s}{s^2+1}\right) + \frac{1}{10}\left(\frac{1}{s^2+1}\right)$

$= \left[-\frac{3}{10}e^{-3t} + \frac{3}{10}\cos t + \frac{1}{10}\sin t\right]$

b. $\frac{e^{-s}-e^{2-2s}}{s-1} = \frac{e^{-s}}{s-1} - \frac{e^2 \cdot e^{-2s}}{s-1}$

$= u(t-1)e^{t-1} - e^2 u(t-2)e^{t-2}$

$= u(t-1)e^{t-1} - u(t-2)e^t$

$\begin{cases} 0 & t<1 \\ e^t & 1 \le t \le 2 \\ e^2 & t>2 \end{cases}$

2. Find the Laplace transform for

a. $f(t) = \frac{1}{3}\int_0^t e^{-2\tau}\sin(3(t-\tau))\,d\tau$

$\frac{1}{3} \cdot \frac{1}{s+2} \cdot \frac{3}{s^2+9} = \frac{1}{(s+2)(s^2+9)}$

b. $g(t) = \delta(t-e) + e^{-t}$

$e^{-es} + \frac{1}{s+1}$

c. $y'' + 6y' - 3y = \sin t, y(0) = 2, y'(0) = 1$

$s^2Y(s) - s(2) - 1 + 6[sY(s) - 2] - 3Y(s) = \frac{1}{s^2+1}$

$[s^2+6s-3]Y(s) - 2s - 1 - 12 = \frac{1}{s^2+1}$

$Y(s) = \frac{(2s+13)(s^2+1)+1}{(s^2+6s-3)(s^2+1)} = \frac{2s^3+13s^2+3s+13}{(s^2+1)(s^2+6s+9-12)} =$

$\frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+6s-3} = \frac{As^3+6As^2-3As+Bs^2+6Bs-3B+Cs^3+Ds^2+Cs+D}{(s^2+1)(s^2+6s+9-12)}$

$(s+3)^2-12$

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$A + C = 2$

$6A + B + D = 13$

$-3A + 6B + C = 3$

$-3B + D = 13$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 6 & 1 & 0 & 1 & 13 \\ -3 & 6 & 1 & 0 & 3 \\ 0 & -3 & 0 & 1 & 13 \end{array}\right]$$

$A = -4/13$

$B = 3/26$

$C = 27/13$

$D = 347/26$

$$-\frac{1}{13}\left(\frac{s}{s^2+1}\right) + \frac{3}{26}\left(\frac{1}{s^2+1}\right) + \frac{27}{13}\left(\frac{s}{(s+3)^2-12}\right) + \frac{347}{26}\left(\frac{1}{(s+3)^2-12}\right)$$

$$+3\left(\frac{27}{13}\right) + -3\left(\frac{27}{13}\right)$$

$$-\frac{1}{13}\left(\frac{s}{s^2+1}\right) + \frac{3}{26}\left(\frac{1}{s^2+1}\right) + \frac{27}{13}\left(\frac{s+3}{(s+3)^2-12}\right) + \frac{185}{26}\left(\frac{\sqrt{12}}{(s+3)^2-12}\right)\frac{}{\sqrt{12}}$$

$$-\frac{1}{13}\cos t + \frac{3}{26}\sin t + \frac{27}{13}e^{-3t}\cosh\left(\sqrt{12}t\right) + \frac{185\sqrt{3}}{156}e^{-3t}\sinh\left(\sqrt{12}t\right)$$

instead of completing the square for s^2+6s-3

you can use the quadratic formula to factor

your solutions will have only e to a messy power in it, but not the same as w/ cosh & sinh

they will be algebraically equivalent but

also harder to find C3D coeffs w/o a computer-algebra system, except as decimals

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	t^p, $p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n+\frac{1}{2}}$, $n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{\left(s^2+a^2\right)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{\left(s^2+a^2\right)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{\left(s^2+a^2\right)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{\left(s^2+a^2\right)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s\left(s^2-a^2\right)}{\left(s^2+a^2\right)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s\left(s^2+3a^2\right)}{\left(s^2+a^2\right)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at}$, $n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			