MAT 212, Quiz #14, Summer 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use series solution methods to solve the equation $2y'' + xy' + 3y = 0, x_0 = 0$. Write at least 4 terms of each solution

$$2\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} + x\sum_{n=1}^{\infty} a_n n x^{n-1} + 3\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} 2a_{n+2}(n+2)(n+1)x^n + \sum_{n=1}^{\infty} a_n n x^n + \sum_{n=0}^{\infty} 3a_n x^n = 0$$

$$\sum_{n=1}^{\infty}\left[2a_{n+2}(n+2)(n+1) + a_n n + 3a_n\right]x^n + 2a_2(2)(1) + 3a_0(1) = 0$$

$$2a_{n+2}(n+2)(n+1) = -a_n(n+3)$$

$$4a_2 = -3a_0$$

$$a_{n+2} = \frac{-a_n(n+3)}{2(n+2)(n+1)}$$

$$a_2 = -\frac{3}{4}a_0$$

$n=1$

$$a_3 = \frac{-a_1(\cancel{4})}{\cancel{2}(3)(\cancel{2})} = \frac{-a_1}{3}$$

$n=2$

$$a_4 = \frac{-a_2(5)}{2(4)(3)} = \frac{-5}{\cancel{24}_{8}}\left(-\frac{\cancel{3}}{4}a_0\right) = \frac{5}{32}a_0$$

$n=3$

$$a_5 = \frac{-a_3(\cancel{6})^3}{\cancel{2}(5)(4)} = \frac{-3}{20}\left(-\frac{a_1}{3}\right) = \frac{a_1}{20}$$

$n=4$

$$a_6 = \frac{-a_4(7)}{2(6)(5)} = \frac{5}{32}\left(\frac{-7}{12\cdot 5}\right)a_0 = \frac{-7}{384}a_0$$

$n=5$

$$a_7 = \frac{-a_5(\cancel{8})^{4\,2}}{\cancel{2}(\cancel{6}_3)(7)} = \frac{a_1}{\cancel{20}_{10}}\left(-\frac{\cancel{2}^1}{21}\right) = \frac{-1}{210}a_1$$

$$y(x) = a_0\left(1 - \frac{3}{4}x^2 + \frac{5}{32}x^4 - \frac{7}{384}x^6 + \dots\right) + a_1\left(x - \frac{1}{3}x^3 + \frac{1}{20}x^5 - \frac{1}{210}x^7 + \dots\right)$$