

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Set up the system of equations to solve the traffic network problem below. Solve the system (you may use your calculator). If the system is dependent, write the solution in parametric form.

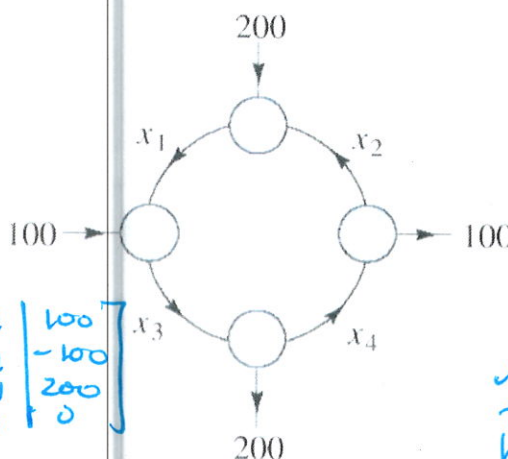
$$\begin{aligned} 200 + x_2 &= x_1 \Rightarrow x_1 - x_2 = 200 \\ x_4 &= 100 + x_2 \Rightarrow -x_2 + x_4 = 100 \\ x_3 &= 200 + x_4 \Rightarrow x_3 - x_4 = 200 \\ 100 + x_1 &= x_3 \Rightarrow -x_1 + x_3 = 100 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 200 \\ 0 & -1 & 0 & 1 & 100 \\ 0 & 0 & 1 & -1 & 200 \\ -1 & 0 & 1 & 0 & 100 \end{array} \right] \xrightarrow{\text{rref}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & -1 & -100 \\ 0 & 0 & 1 & -1 & 200 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 - x_4 &= 100 \Rightarrow x_1 = x_4 + 100 \\ x_2 - x_4 &= -100 \Rightarrow x_2 = x_4 - 100 \\ x_3 - x_4 &= 200 \Rightarrow x_3 = x_4 + 200 \\ x_4 &= x_4 \Rightarrow x_4 = x_4 \end{aligned}$$

$x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 100 \\ -100 \\ 200 \\ 0 \end{bmatrix}$$



note: since traffic can't be negative x_4 must be ≥ 0

2. For the matrices $A = \begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}$.

a. Find $B + \frac{1}{2}A$

$$\begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1/2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 3/2 & 0 \end{bmatrix}$$

b. Find AB

$$\begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 8-4 & 2-4 \\ -4+8 & -1-8 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 4 & -9 \end{bmatrix}$$