202 Homework #9 key

1a. $A-\lambda I = \begin{bmatrix} 1-\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(3-\lambda)+2 = \lambda^2-4\lambda+3+2 = \lambda^2-4\lambda+5=0$

$\lambda = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i$

$\lambda_1 = 2+i$

$\begin{bmatrix} 1-2-i & -2 \\ 1 & 3-2-i \end{bmatrix} = \begin{bmatrix} -1-i & -2 \\ 1 & 1-i \end{bmatrix}$

$x_1 + (1-i)x_2 = 0$
$x_1 = -(1-i)x_2$
$x_2 = x_2$

$\vec{v}_1 = \begin{bmatrix} 1-i \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$

$\lambda_2 = 2-i \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$

$\lambda = a - bi$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \quad PCP^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 0-\lambda & 5 \\ -2 & 2-\lambda \end{bmatrix} \Rightarrow (-\lambda)(2-\lambda)+10=0$

$\lambda^2-2\lambda+10=0$

$\lambda = \frac{2 \pm \sqrt{4-40}}{2} = \frac{2 \pm 6i}{2} = 1 \pm 3i$

$\lambda_1 = 1+3i$

$\begin{bmatrix} -1-3i & 5 \\ -2 & 2-1-3i \end{bmatrix} = \begin{bmatrix} -1-3i & 5 \\ -2 & 1-3i \end{bmatrix}$

$-2x_1 + (1-3i)x_2 = 0$
$x_1 = \frac{1-3i}{2} x_2$
$x_2 = x_2$

$\vec{v}_1 = \begin{bmatrix} 1-3i \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} i$

$\lambda_2 = 1-3i \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \end{bmatrix} i$

$P = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -3-\lambda & -8 \\ 4 & 5-\lambda \end{bmatrix} \Rightarrow (-3-\lambda)(5-\lambda)+32 = \lambda^2-2\lambda-15+32 = \lambda^2-2\lambda+17=0$

$\lambda = \frac{2 \pm \sqrt{4-68}}{2} = \frac{2 \pm 8i}{2} = 1 \pm 4i$

$\lambda_1 = 1+4i$

$\begin{bmatrix} -3-1-4i & -8 \\ 4 & 5-1-4i \end{bmatrix} = \begin{bmatrix} -4-4i & -8 \\ 4 & 4-4i \end{bmatrix} \div 4$

$x_1 + (1-i)x_2 = 0$
$x_1 = -(1-i)x_2$
$x_2 = x_2$

$\vec{v}_1 = \begin{bmatrix} 1-i \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$

$\lambda_2 = 1-4i \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -4 \\ 4 & 1 \end{bmatrix}$

d. $\begin{bmatrix} -3-\lambda & 7 \\ 5 & -1-\lambda \end{bmatrix} \Rightarrow (-3-\lambda)(-1-\lambda)-35 = \lambda^2+4\lambda+3-35 = \lambda^2+4\lambda-32=0$

$(\lambda+8)(\lambda-4)=0 \quad \lambda = -8, 4$

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1d cont'd.

$\lambda_1 = -8$

$$\begin{bmatrix} -3+8 & 7 \\ 5 & -1+8 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 5 & 7 \end{bmatrix} \qquad 5x_1 = -7x_2 \quad x_1 = -\tfrac{7}{5}x_2 \quad x_2 = x_2 \qquad \vec{v}_1 = \begin{bmatrix} -7 \\ 5 \end{bmatrix}$$

$\lambda_2 = 4$

$$\begin{bmatrix} -3-4 & 7 \\ 5 & -1-4 \end{bmatrix} = \begin{bmatrix} -7 & 7 \\ 5 & -5 \end{bmatrix} \div 5 \qquad x_1 - x_2 = 0 \quad x_1 = x_2 \quad x_2 = x_2 \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} -7 & 1 \\ 5 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} -8 & 0 \\ 0 & 4 \end{bmatrix}$$

e. $$\begin{bmatrix} -4-\lambda & 1 \\ 6 & -5-\lambda \end{bmatrix} \Rightarrow (-4-\lambda)(-5-\lambda) - 6 = \lambda^2 + 9\lambda + 20 - 6 = \lambda^2 + 9\lambda + 14 = 0$$

$$(\lambda+7)(\lambda+2) = 0 \quad \lambda = -7, -2$$

$\lambda_1 = -7$

$$\begin{bmatrix} -4+7 & 1 \\ 6 & -5+7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \qquad 3x_1 + x_2 = 0 \quad x_1 = -\tfrac{1}{3}x_2 \quad x_2 = x_2 \qquad \vec{v}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$\lambda_2 = -2$

$$\begin{bmatrix} -4+2 & 1 \\ 6 & -5+2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 6 & -3 \end{bmatrix} \qquad -2x_1 + x_2 = 0 \quad x_1 = \tfrac{1}{2}x_2 \quad x_2 = x_2 \qquad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$P = \begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} -7 & 0 \\ 0 & -2 \end{bmatrix}$$

f. $$\begin{bmatrix} 3-\lambda & -2 \\ 2 & 3-\lambda \end{bmatrix} \Rightarrow (3-\lambda)^2 + 4 = \lambda^2 - 6\lambda + 9 + 4 = 0 \qquad \lambda^2 - 6\lambda + 13 = 0$$

$$\lambda = \frac{6 \pm \sqrt{36 - 52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$\lambda_1 = 3+2i$

$$\begin{bmatrix} 3-3-2i & -2 \\ 2 & 3-3-2i \end{bmatrix} = \begin{bmatrix} -2i & -2 \\ 2 & -2i \end{bmatrix} \qquad \cancel{2}x_1 = \cancel{2}ix_2 \quad x_2 = x_2 \qquad \vec{v}_1 = \begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$$

$$\lambda_2 = 3 - 2i \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$$

$$P = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$$

g. $$\begin{bmatrix} -4-\lambda & 5 \\ -5 & -4-\lambda \end{bmatrix} \Rightarrow (-4-\lambda)(-4-\lambda)^2 + 25 = \lambda^2 + 8\lambda + 16 + 25 = \lambda^2 + 8\lambda + 41 = 0$$

$$\lambda = \frac{-8 \pm \sqrt{64 - 164}}{2} = \frac{-8 \pm 10i}{2} = -4 \pm 5i$$

1g cont'd

$\lambda_1 = -4+5i$

$$\begin{bmatrix} -4+4-5i & 5 \\ -5 & -4+4-5i \end{bmatrix} = \begin{bmatrix} -5i & 5 \\ -5 & -5i \end{bmatrix}$$

$-5x_1 - (5i)x_2 = 0$

$x_1 = \dfrac{5i}{-5} x_2$

$x_2 = x_2$

$$\vec{v_1} = \begin{bmatrix} 5i \\ -5 \end{bmatrix} = \begin{bmatrix} i \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} i$$

$\lambda_2 = -4-5i$

$$\vec{v_2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} i$$

$$P = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -4 & -5 \\ 5 & -4 \end{bmatrix}$$

h. $\begin{bmatrix} -2-\lambda & 2 \\ -5 & 6-\lambda \end{bmatrix} \Rightarrow (-2-\lambda)(6-\lambda)+10 = \lambda^2-4\lambda-12+10 = \lambda^2-4\lambda-2=0$

$$\lambda = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$$

$\lambda_1 = 2+\sqrt{6}$

$$\begin{bmatrix} -2-2-\sqrt{6} & 2 \\ -5 & 6-2-\sqrt{6} \end{bmatrix} = \begin{bmatrix} -4-\sqrt{6} & 2 \\ -5 & 4-\sqrt{6} \end{bmatrix}$$

$-5x_1 + (4-\sqrt{6})x_2$

$x_1 = \dfrac{4-\sqrt{6}}{5} x_2$

$x_2 = x_2$

$$\vec{v_1} = \begin{bmatrix} 4-\sqrt{6} \\ 5 \end{bmatrix}$$

$\lambda_2 = 2-\sqrt{6}$

$$\vec{v_2} = \begin{bmatrix} 4+\sqrt{6} \\ 5 \end{bmatrix}$$

$$P = \begin{bmatrix} 4-\sqrt{6} & 4+\sqrt{6} \\ 5 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2+\sqrt{6} & 0 \\ 0 & 2-\sqrt{6} \end{bmatrix}$$

i. $\begin{bmatrix} -2-\lambda & 5 & 3 \\ 0 & 2-\lambda & -4 \\ 0 & -1 & 2-\lambda \end{bmatrix} = (-2-\lambda)\begin{vmatrix} 2-\lambda & -4 \\ -1 & 2-\lambda \end{vmatrix} = (-2-\lambda)\left[(2-\lambda)^2-4\right] =$

$(-2-\lambda)(\lambda^2-4\lambda+4-4) = (-2-\lambda)(\lambda^2-4\lambda) =$

$(-2-\lambda)(\lambda)(\lambda-4) = 0 \quad \lambda = -2, 0, 4$

$\lambda_1 = -2$

$$\begin{bmatrix} -2+2 & 5 & 3 \\ 0 & 4 & -4 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ 0 & 4 & -4 \\ 0 & -1 & 0 \end{bmatrix} \overset{\text{rref}}{\Rightarrow} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_1$

$x_2 = 0$

$x_3 = 0$

$$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_2 = 0$

$$\begin{bmatrix} -2 & 5 & 3 \\ 0 & 2 & -4 \\ 0 & -1 & 2 \end{bmatrix} \overset{\text{rref}}{\Rightarrow} \begin{bmatrix} 1 & 0 & -13/2 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 13/2\, x_3$

$x_2 = 2x_3$

$x_3 = x_3$

$$\Rightarrow \vec{v_2} = \begin{bmatrix} 13 \\ 4 \\ 2 \end{bmatrix}$$

2a cont'd

$$P=\begin{bmatrix}1&0\\3&1\end{bmatrix}\quad D=\begin{bmatrix}1&0\\1&-1\end{bmatrix}\quad D^k=\begin{bmatrix}1&0\\0&(-1)^k\end{bmatrix}$$

$$\begin{bmatrix}1&0\\3&1\end{bmatrix}\begin{bmatrix}1&0\\0&(-1)^k\end{bmatrix}\begin{bmatrix}1&0\\-3&1\end{bmatrix}=\begin{bmatrix}1&0\\3&(-1)^k\end{bmatrix}\begin{bmatrix}1&0\\-3&1\end{bmatrix}=\begin{bmatrix}1&0\\3-3(-1)^k&(-1)^k\end{bmatrix}$$
$$=A^k$$

b. $$\begin{bmatrix}1-\lambda&3\\4&2-\lambda\end{bmatrix}\Rightarrow(1-\lambda)(2-\lambda)-12=\lambda^2-3\lambda+2-12=\lambda^2-3\lambda-10=0$$
$$(\lambda-5)(\lambda+2)=0\quad\lambda=5,-2$$

$\lambda_1=5$

$$\begin{bmatrix}-4&3\\4&-3\end{bmatrix}\quad 4x_1=3x_2,\ x_1=\tfrac{3}{4}x_2,\ x_2=x_2\quad\vec{v}_1=\begin{bmatrix}3\\4\end{bmatrix}$$

$x_2=-2$

$$\begin{bmatrix}3&3\\4&4\end{bmatrix}\quad x_1=-x_2,\ x_2=x_2\quad\vec{v}_2=\begin{bmatrix}-1\\1\end{bmatrix}$$

$$P=\begin{bmatrix}3&-1\\4&1\end{bmatrix}\quad D=\begin{bmatrix}5&0\\0&-2\end{bmatrix}\quad D^k=\begin{bmatrix}5^k&0\\0&(-2)^k\end{bmatrix}$$

$$\begin{bmatrix}3&-1\\4&1\end{bmatrix}\begin{bmatrix}5^k&0\\0&(-2)^k\end{bmatrix}\begin{bmatrix}1/7&1/7\\-4/7&3/7\end{bmatrix}=\begin{bmatrix}3(5)^k&-(-2)^k\\4(5)^k&(-2)^k\end{bmatrix}\begin{bmatrix}1/7&1/7\\-4/7&3/7\end{bmatrix}=$$

$$\begin{bmatrix}\frac{1}{7}(3\cdot5^k+4(-2)^k)&\frac{3}{7}(5^k-(-2)^k)\\\frac{4}{7}(5^k-(-2)^k)&\frac{1}{7}(4\cdot5^k+3(-2)^k)\end{bmatrix}=A^k$$

c. $$\begin{bmatrix}2-\lambda&2&-1\\1&3-\lambda&-1\\-1&-2&2-\lambda\end{bmatrix}\quad\lambda=1,5$$

$$\begin{bmatrix}1&2&-1\\1&2&-1\\-1&-2&1\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&2&-1\\0&0&0\\0&0&0\end{bmatrix}\quad\begin{aligned}x_1&=-2x_2+x_3\\x_2&=x_2\\x_3&=x_3\end{aligned}$$

$$\vec{v}_1=\begin{bmatrix}-2\\1\\0\end{bmatrix}\quad\vec{v}_2=\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}-3&2&-1\\1&-2&-1\\-1&-2&-3\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&1\\0&1&1\\0&0&0\end{bmatrix}\quad\begin{aligned}x_1&=-x_3\\x_2&=-x_3\\x_3&=x_3\end{aligned}\quad\vec{v}_3=\begin{bmatrix}-1\\-1\\1\end{bmatrix}$$

$$P=\begin{bmatrix}-2&1&-1\\1&0&-1\\0&1&1\end{bmatrix}\quad D=\begin{bmatrix}1&0&0\\0&1&0\\0&0&5\end{bmatrix}\quad D^k=\begin{bmatrix}1&0&0\\0&1&0\\0&0&5^k\end{bmatrix}$$

$$\begin{bmatrix}-2&1&-1\\1&0&-1\\0&1&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&0&5^k\end{bmatrix}\begin{bmatrix}-1/4&1/2&1/4\\1/4&1/2&3/4\\-1/4&-1/2&1/4\end{bmatrix}=\begin{bmatrix}-2&1&-5^k\\1&0&-5^k\\0&1&5^k\end{bmatrix}\begin{bmatrix}-1/4&1/2&1/4\\1/4&1/2&3/4\\-1/4&-1/2&1/4\end{bmatrix}=$$

2c. cont'd

$$\begin{bmatrix} \frac{3+5^k}{4} & \frac{1+5^k}{2} & \frac{1-5^k}{4} \\ \frac{-1+5^k}{4} & \frac{1+5^k}{2} & \frac{1-5^k}{4} \\ \frac{1-5^k}{4} & \frac{1-5^k}{2} & \frac{3+5^k}{4} \end{bmatrix}$$

d. $\begin{bmatrix} 2-\lambda & -2 & -2 \\ 3 & -3-\lambda & -2 \\ 2 & -2 & -2-\lambda \end{bmatrix}$

$\lambda = -2, -1, 0$

$\lambda_1 = -2$ $\begin{bmatrix} 4 & -2 & -2 \\ 3 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{matrix} x_1 = x_3 \\ x_2 = x_3 \\ x_3 = x_3 \end{matrix}$ $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -1$

$\begin{bmatrix} 3 & -2 & -2 \\ 3 & -2 & -2 \\ 2 & -2 & -1 \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{matrix} x_1 = x_3 \\ x_2 = 1/2\, x_3 \\ x_3 = x_3 \end{matrix}$ $\vec{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

$\lambda_3 = 0$

$\begin{bmatrix} 2 & -2 & -2 \\ 3 & -3 & -2 \\ 2 & -2 & -2 \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{matrix} x_1 = x_2 \\ x_2 = x_2 \\ x_3 = 0 \end{matrix}$ $\vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $D^k = \begin{bmatrix} (-2)^k & 0 & 0 \\ 0 & (-1)^k & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} (-2)^k & 0 & 0 \\ 0 & (-1)^k & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} (-2)^k & 2(-1)^k & 0 \\ (-2)^k & (-1)^k & 0 \\ (-2)^k & 2(-1)^k & 0 \end{bmatrix}\begin{bmatrix} -2 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)^{k+1} + 2(-1)^k & 2(-2)^k - 2(-1)^k & (-2)^k \\ (-2)^{k+1} + (-1)^k & 2(-2)^k + (-1)^{k+1} & (-2)^k \\ (-2)^{k+1} + 2(-1)^k & 2(-2)^k - 2(-1)^k & (-2)^k \end{bmatrix} = A^k$$

3. no. each eigenvalue comes with a one-dimensional eigenspace at least, which accounts for 3 of 4 dimensions, but since one is 2-dimensional which adds one more, giving us 4 vectors and diagonalizability.

4. $\begin{bmatrix} 1-\lambda & 2 \\ 3 & -4-\lambda \end{bmatrix} \Rightarrow (1-\lambda)(-4-\lambda)-6 = \lambda^2+3\lambda-4-6 = \lambda^2+3\lambda-10=0$

$(\lambda+5)(\lambda-2)=0 \quad \lambda=-5, 2$

$\lambda_1=-5 \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$ $\quad 3x_1=-x_2$, $x_1=-\frac{1}{3}x_2$, $x_2=x_2$ $\quad \vec{v}_1=\begin{bmatrix} -1 \\ 3 \end{bmatrix}$ $\quad \lambda_2=2 \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix}$ $\quad x_1=2x_2$, $x_2=x_2$ $\quad \vec{v}_2=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$B=\left\{\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\} \quad D=[T]_B=\begin{bmatrix} -5 & 0 \\ 0 & 2 \end{bmatrix}$

5. $A\vec{x}=\lambda x$

a. $2A\vec{x}=(2\lambda)\vec{x}$ $\quad -6, 8$

b. $5A\vec{x}=(5\lambda)\vec{x}$ $\quad -15, 20$

c. $(A-3I)\vec{x}=\lambda\vec{x}-3I\vec{x}=(\lambda-3)\vec{x}$ $\quad -6, 1$

d. $(A+4I)\vec{x}=\lambda\vec{x}+4\vec{x}=(\lambda+4)\vec{x}$ $\quad 1, 8$

e. $(2A)\vec{x}=(2\lambda)\vec{x}$
$+5Ix \quad +5I\vec{x}$ $\Rightarrow (2A+5I)\vec{x}=2\lambda\vec{x}+5\vec{x}=(2\lambda+5)\vec{x}$

$-1, 13$

6. a. true

b. false. invertible only if 0 is <u>not</u> an eigenvalue of A

c. false (they may or may not)

d. false. only if triangular or diagonal

e. false

f. false only if triangular or diagonal

g. false. Some do.

h. false $\lambda=-5$

i. true

j. false. 0 may be an eigenvalue