202 Homework #8 key

1a. $A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Counterclockwise

b. $A = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2a. $p(t) = at^2 + bt + c$

$(t+3)(at^2+bt+c) = at^3 + bt^2 + ct + 3at^2 + 3bt + 3c$

b. It is linear since
i) $(t+3)p(t) = \vec{0}$ when $p(t) = \vec{0}$
ii) $(t+3)p(t) + q(t)(t+3) = (t+3)(p(t)+q(t))$
iii) $k(t+3)p(t) = (t+3)[k\,p(t)]$.
and its written as a matrix, so its linear.

c. $\begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

3a. $\begin{bmatrix} 2 & -4 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 2 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

c. $(2t^2 - t + 6)(a_0 + a_1 t + a_2 t^2 + a_3 t^3) = 6a_0 + 6a_1 t + 6a_2 t^2 + 6a_3 t^3 - a_0 t - a_1 t^2 - a_2 t^3 - a_3 t^4 + 2a_0 t^2 + 2a_1 t^3 + 2a_2 t^4 + 2a_3 t^5$

$\begin{bmatrix} 6 & 0 & 0 & 0 \\ -1 & 6 & 0 & 0 \\ 2 & -1 & 6 & 0 \\ 0 & 2 & -1 & 6 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

2d. $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \Rightarrow$

$p'(t) = a_1 + 2a_2 t + 3a_3 t^2$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

2e. $a_1 e^x + a_2 e^{-x} + a_3 e^{5x} + a_4 e^{-7x} = y(x)$

$y'(x) = a_1 e^x + (-1) a_2 e^{-x} + 5a_3 e^{5x} - 7a_4 e^{-7x}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

f. $y(x) = a_1 e^{3x} \cos(2x) + a_2 e^{3x} \sin(2x)$

$y'(x) = a_1 3e^{3x} \cos 2x + (-2) a_1 e^{3x} \sin 2x + 3a_2 e^{3x} \sin 2x + 2a_2 e^{3x} \cos 2x$

$$\begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$$

g. $225° = 5\pi/4$ $\quad \begin{bmatrix} \cos(5\pi/4) & -\sin(5\pi/4) \\ \sin(5\pi/4) & \cos(5\pi/4) \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

h.
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{3}/2 & -1/2 \\ 0 & -1 & \sqrt{3} \\ 4 & 0 & 0 \end{bmatrix}$$

4a. $P_C^{-1} P_B = \begin{bmatrix} 13/9 & -14/9 \\ -2/9 & -2/9 \end{bmatrix}$

b. $P_C^{-1} P_B = \begin{bmatrix} 5/8 & 1/2 & -5/8 \\ 17/8 & -1/2 & -17/8 \\ 1/8 & 1/2 & 7/8 \end{bmatrix}$

c. $P_C^{-1} P_B = \begin{bmatrix} -11/17 & 8/17 & 22/17 & 25/17 \\ -12/17 & 1/17 & -10/17 & -50/17 \\ 22/17 & 35/17 & 7/17 & 1/17 \\ 7/17 & -2/17 & -14/17 & -2/17 \end{bmatrix}$

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5a. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

b. $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

6a. $P^{-1}AP = \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 12 & 7 \\ -20 & -11 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ -8 & -4 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & 0 \end{bmatrix}$
$= B$

b. $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 2 & 3 \end{bmatrix}$
$= B$

7a. $B = P^{-1}AP \Rightarrow PBP^{-1} = A$

$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & -15 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix}$ $\quad B^4 = P^{-1}A^4P = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -74 & -225 \\ 30 & 91 \end{bmatrix}$

b. $B = P^{-1}AP \Rightarrow PBP^{-1} = A$

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$e^A = \begin{bmatrix} e^4 & 0 & 0 \\ 0 & e^{-2} & 0 \\ 0 & 0 & e^{-2} \end{bmatrix}$ $\quad e^B = P^{-1}e^AP = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} e^4 & 0 & 0 \\ 0 & e^{-2} & 0 \\ 0 & 0 & e^{-2} \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1/2e^4 & 1/2e^{-2} & 0 \\ 1/2e^4 & -1/2e^{-2} & 0 \\ 0 & 0 & e^{-2} \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2e^4 + 1/2e^{-2} & 1/2e^4 - 1/2e^{-2} & 0 \\ 1/2e^4 - 1/2e^{-2} & 1/2e^4 + 1/2e^{-2} & 0 \\ 0 & 0 & e^{-2} \end{bmatrix}$