202 Homework #11 key

1a. false. the best approximation is $\text{proj}_W \vec{y}$.

b. true

c. true

d. true

e. true

2. a. $A^TA\vec{x} = A^T\vec{b} \Rightarrow \vec{x} = (A^TA)^{-1}A^T\vec{b}$

no unique least squares solution since the columns of A are not independent. (A^TA) is not invertible

row reducing $A^TA\vec{x} = A^T\vec{b} \Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 2 & 14 \\ 2 & 2 & 0 & 4 \\ 2 & 0 & 2 & 10 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = -x_3 + 5$
$x_2 = x_3 - 3$
$x_3 = x_3$

$\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} t + \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}$

b. $A^TA\vec{x} = A^Tb \Rightarrow \vec{x} = (A^TA)^{-1}A^T\vec{b} \Rightarrow \vec{x} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$

3a. $\beta_0 + \beta_1 x = y$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ $(A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} -.6 \\ .7 \end{bmatrix}$

$y = -.6 + .7x$

b. $\beta_0 + \beta_1 x + \beta_2 x^2 = y$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ $(A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} -.6 \\ .7 \\ 1.5\times10^{-12} \end{bmatrix}$ ← since this is nearly zero, a quadratic model is not appropriate

$y = -.6 + .7x + (1.5\times10^{-12})x^2$

3c. $\beta_1 x + \beta_2 x^2 + \beta_3 x^3 = y$

$$A = \begin{bmatrix} 4 & 16 & 64 \\ 6 & 36 & 216 \\ 8 & 64 & 512 \\ 10 & 100 & 1000 \\ 12 & 144 & 12^3 \\ 14 & 196 & 14^3 \\ 16 & 256 & 16^3 \\ 18 & 18^2 & 18^3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1.58 \\ 2.08 \\ 2.5 \\ 2.8 \\ 3.1 \\ 3.4 \\ 3.8 \\ 4.32 \end{bmatrix} \quad (A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} .5132 \\ -.0335 \\ .0010 \end{bmatrix}$$

$$y = .5132x - .0335x^2 + .0010x^3$$

4. when you plug in values of x, you end up w/ a linear equation for A & B. But when we plug in values for x in $A\sin(Bx+C)+D$ B, in particular, remains nonlinear.
(we can eliminate C by rewriting w/ cosine, but the frequency of the functions needs to be specified in advance to make this linear).

5. $y = a\ln(bx)$, $y = ae^{bx}$ or $y = a(b^x)$, $y = \frac{a}{x} + b$
$y = a\ln x + b$, any polynomial, $y = ae^x + be^{-x}$
$y = a\cos x + b\sin x$, $y = a\tan x + b\sec x$

answers will vary.