

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the determinant of the matrix

$$\begin{bmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{bmatrix} \text{ by}$$

a. The cofactor method.

$$2 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 8 \end{vmatrix} = 2(5) \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix} = 10(7-6) = 10(1) = 10$$

b. By row-reducing. Verify that the values are the same in both cases.

$$\begin{array}{l} R_2 \leftrightarrow R_1 \\ (-1) \end{array} \begin{vmatrix} 1 & 7 & 2 & -5 \\ 6 & 0 & 0 & 5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix} \begin{array}{l} -6R_1 + R_2 \rightarrow R_2 \\ -2R_1 + R_3 \rightarrow R_3 \\ -8R_1 + R_4 \rightarrow R_4 \end{array} \begin{array}{l} (1) \end{array} \begin{vmatrix} 1 & 7 & 2 & -5 \\ 0 & -42 & -12 & 35 \\ 0 & -14 & -4 & 10 \\ 0 & -53 & -15 & 48 \end{vmatrix} \begin{array}{l} -1/42 R_2 \rightarrow R_2 \\ (42) \end{array}$$

$$\begin{vmatrix} 1 & 7 & 2 & -5 \\ 0 & 1 & 2/7 & -5/6 \\ 0 & -14 & -4 & 10 \\ 0 & -53 & -15 & 48 \end{vmatrix} \begin{array}{l} 14R_2 + R_3 \rightarrow R_3 \\ 53R_2 + R_4 \rightarrow R_4 \end{array} \begin{array}{l} (1) \end{array} \begin{vmatrix} 1 & 7 & 2 & -5 \\ 0 & 1 & 2/7 & -5/6 \\ 0 & 0 & 0 & -5/3 \\ 0 & 0 & 1/7 & -265/6 \end{vmatrix} \begin{array}{l} R_3 \leftrightarrow R_4 \\ (-1) \end{array}$$

$$\begin{vmatrix} 1 & 7 & 2 & -5 \\ 0 & 1 & 2/7 & -5/6 \\ 0 & 0 & 1/7 & -265/6 \\ 0 & 0 & 0 & -5/3 \end{vmatrix} \quad (-1)(-42)(-1) \left[(1)(1) \left(\frac{1}{7}\right) (-5/3) \right]$$

$$-42 \left(-\frac{5}{21}\right) = 2(5) = 10$$