Math 2568, Quiz #12, Summer 2015 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Solve the discrete dynamical system given by $\vec{x}_{k+1} = \begin{bmatrix} .38 & .24 \\ -.36 & 1.22 \end{bmatrix} \vec{x}_k$. Sketch a graph of the eigenvectors and plot some sample trajectories. Is the origin an attractor, a repeller or a saddle point? Write the solution as $\vec{x}_k = c_1\lambda_1^k\vec{v}_1 + c_2\lambda_2^k\vec{v}_2$.

$$\begin{vmatrix} .38-\lambda & .24 \\ -.36 & 1.22-\lambda \end{vmatrix} = (.38-\lambda)(1.22-\lambda) + (.24)(.36) = 0$$

$$\lambda^2 - 1.6\lambda + .55 = 0$$

$$\lambda_1 = .5 \qquad \lambda_2 = 1.1$$

$\lambda_1 = .5$ $\begin{bmatrix} .38-.5 & .24 \\ -.36 & 1.22-.5 \end{bmatrix} = \begin{bmatrix} -.12 & .24 \\ -.36 & .72 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \Rightarrow x_1 - 2x_2 = 0$

$\Rightarrow x_1 = 2x_2$ $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$x_2 = x_2$

$\lambda_2 = 1.1$ $\begin{bmatrix} .38-1.1 & .24 \\ -.36 & 1.22-1.1 \end{bmatrix} = \begin{bmatrix} -.72 & .24 \\ -.36 & .12 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix} \Rightarrow 3x_1 - x_2 = 0$

$\Rightarrow x_1 = \frac{1}{3}x_2$ $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$x_2 = x_2$

$$\vec{x}_k = c_1(.5)^k \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2(1.1)^k \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Origin is a saddle point