Math 2415, Quiz #8, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of 10 cm/sec, and if there is no damping, determine the position y of the mass at any time t. When does the mass first return to equilibrium? (i.e. when is y=0?)

$M = 100g = .1\ kg$ $\quad 5\ cm = .05\ m$

$.1(98) = F = k(.05)$ $\quad y'(0) = -10\ \frac{cm}{sec} = -.1\ m/sec$ $\quad y(0) = 0$

$\frac{.98}{.05} = k = 19.6$ $\quad \gamma = 0$

$my'' + \gamma y' + ky = 0$

$.1y'' + 0y' + 19.6y = 0$

$\frac{.1y''}{.1} + \frac{19.6y}{.1} = 0$

$y'' + 196y = 0$

$r^2 + 196 = 0$

$r = \pm 14i$

$y = c_1 \cos(14t) + c_2 \sin(14t)$

$y(0) = 0 = c_1(1) + c_2(0)$

$\Rightarrow c_1 = 0$

$y(t) = c_2 \sin(14t)$

$y'(t) = c_2 \cos(14t) * 14$

$y'(0) = -.1 = 14c_2 \cos(0) = 14c_2(1)$

$= -\frac{1}{140} = c_2$

$y(t) = -\frac{1}{140} \sin(14t)$

$0 = -\frac{1}{140} \sin(14t)$

$\sin(14t) = 0 \Rightarrow 14t = \pi \Rightarrow t = \frac{\pi}{14} \approx .224$ seconds