Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Use the method of integrating factors to solve the ODE $y' - 2y = 3e^t$.

$$\mu = e^{\int -2\,dt} = e^{-2t}$$

$$e^{-2t}y' - 2e^{-2t}y = 3e^t \cdot e^{-2t}$$

$$\int (e^{-2t}y)' = \int 3e^{-t}$$

$$\left(e^{-2t}y = -3e^{-t} + C \right)e^{2t}$$

$$\boxed{y = -3e^t + Ce^{2t}}$$

2. Use the method of integrating factors to solve the ODE $t^3 y' + 4t^2 y = e^{-t}, y(-1) = 0, t < 0$.

$$y' + \frac{4}{t}y = t^{-3}e^{-t}$$

$$\mu = e^{\int \frac{4}{t}dt} = e^{4\ln t} = e^{\ln t^4} = t^4$$

$$t^4 y' + 4t^3 y = te^{-t}$$

$$\int (t^4 y)' = \int te^{-t} \qquad \begin{array}{ll} u=t & dv=e^{-t} \\ du=dt & v=-e^{-t} \end{array}$$

$$t^4 y = -te^{-t} - \int -e^{-t}\,dt$$

$$t^4 y = -te^{-t} - e^{-t} + C$$

$$\rightarrow y = -t^{-3}e^{-t} - t^{-4}e^{-t} + Ct^{-4}$$

$$0 = -(-1)^{-3}e^1 - (-1)^{-4}e^1 + C(-1)^{-4}$$

$$0 = e - e + C$$

$$0 = C$$

$$\therefore \boxed{y = -t^{-3}e^{-t} + t^{-4}e^{-t}}$$