Math 2415, Quiz #12, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. The solution to the heat equation is given by $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2\alpha^2 t}{L^2}} \sin\left(\frac{n\pi x}{L}\right)$ where $c_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$. Find the solution for the case when the rod is 100 cm long and both ends are maintained at 0°C for all $t > 0$. Suppose the initial temperature distribution is given by $u(x,0) = \begin{cases} 0, & 0 \le x < 15 \\ x+20, & 15 \le x < 35 \\ 0, & 85 \le x \le 100 \end{cases}$. Assume that $\alpha^2 = 1$.

$$c_n = \frac{2}{100}\int_{15}^{35}(x+20)\sin\left(\frac{n\pi x}{100}\right)dx =$$

$u = x+20$ $\quad dv = \sin\left(\frac{n\pi x}{100}\right)dx$

$du = dx$ $\quad v = -\frac{100}{n\pi}\cos\left(\frac{n\pi x}{100}\right)$

$$\frac{1}{50}\left[-\frac{(x+20)100}{n\pi}\cos\left(\frac{n\pi x}{100}\right) + \int\frac{100}{n\pi}\cos\left(\frac{n\pi x}{100}\right)dx\right] = \frac{1}{50}\left[-\frac{100(x+20)}{n\pi}\cos\left(\frac{n\pi x}{100}\right) + \frac{100\cdot 100}{n^2\pi^2}\sin\left(\frac{n\pi x}{100}\right)\right]_{15}^{35}$$

$$\frac{1}{50}\left[-\frac{100(55)}{n\pi}\cos\left(\frac{n\pi\cdot 35}{100}\right) + \frac{100^2}{n^2\pi^2}\sin\left(\frac{n\pi\cdot 35}{100}\right) + \frac{100(35)}{n\pi}\cos\left(\frac{n\pi\cdot 15}{100}\right) - \frac{100^2}{n^2\pi^2}\sin\left(\frac{n\pi\cdot 15}{100}\right)\right]$$

$$\frac{1}{50}\left[-\frac{5500}{n\pi}\cos\left(\frac{7n\pi}{20}\right) + \frac{3500}{n\pi}\cos\left(\frac{3n\pi}{20}\right) + \frac{100^2}{n^2\pi^2}\sin\left(\frac{7n\pi}{20}\right) - \frac{100^2}{n^2\pi^2}\sin\left(\frac{3n\pi}{20}\right)\right] = c_n$$

$$u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2 t}{100^2}}\sin\left(\frac{n\pi x}{100}\right)$$