Math 2415, Quiz #10, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the Fourier series for the function $f(x) = \begin{cases} x, & -1 \le x < 0 \\ 2, & 0 \le x < 1 \end{cases}, f(x+2) = f(x)$. Be sure to simplify the coefficients as much as possible.

$$\frac{1}{2}\int_{-1}^{1} f(x)\,dx = \frac{1}{2}\int_{-1}^{0} x\,dx + \frac{1}{2}\int_{0}^{1} 2\,dx = \frac{1}{2}\cdot\frac{1}{2}x^2\Big|_{-1}^{0} + \frac{1}{2}2x\Big|_{0}^{1} = \frac{1}{4}(-1)+1 = \frac{3}{4}$$

$$\int_{-1}^{1} f(x)\sin n\pi x\,dx = \int_{-1}^{0} x\sin n\pi x\,dx + \int_{0}^{1} 2\sin n\pi x\,dx$$

$u = x \quad dv = \sin n\pi x$

$du = dx \quad v = -\frac{1}{n\pi}\cos n\pi x$

$$-\frac{2}{n\pi}\cos n\pi x\Big|_0^1 = -\frac{2}{n\pi}\cos n\pi - \frac{2}{n\pi}$$

$$-\frac{x}{n\pi}\cos n\pi x + \frac{1}{n\pi}\int \cos n\pi x =$$

$$-\frac{x}{n\pi}\cos n\pi x + \frac{1}{n^2\pi^2}\sin n\pi x\Big|_{-1}^{0} = 0+0+\frac{-1}{n\pi}\underbrace{\cos n\pi}_{(-1)^n} - \frac{1}{n^2\pi^2}(0)$$

$$\frac{(-1)^{n+1}}{n\pi} + \frac{2(-1)^{n+1}}{n\pi} - \frac{2}{n\pi} = \boxed{\frac{3(-1)^{n+1}}{n\pi} - \frac{2}{n\pi}}$$

$$\int_{-1}^{1} f(x)\cos n\pi x\,dx = \int_{-1}^{0} x\cos n\pi x\,dx + \int_{0}^{1} 2\cos n\pi x\,dx =$$

$u = x \quad dv = \cos n\pi x\,dx$

$du = dx \quad v = \frac{1}{n\pi}\sin n\pi x$

$$\frac{2}{n\pi}\sin n\pi x\Big|_0^1 = 0$$

$$\frac{x}{n\pi}\sin n\pi x - \int_{-1}^{0}\frac{1}{n\pi}\sin n\pi x\,dx =$$

$$\frac{x}{n\pi}\sin n\pi x + \frac{1}{n^2\pi^2}\cos n\pi x\Big|_{-1}^{0} = 0 + \frac{1}{n^2\pi^2}(1) - 0 - \frac{1}{n^2\pi^2}\cos n\pi = \frac{1}{n^2\pi^2} - \frac{1}{n^2\pi^2}(-1)^n$$

$$f(x) = \frac{3}{4} + \sum_{n=1}^{\infty}\left\{\left[\frac{3(-1)^{n+1}}{n\pi} - \frac{2}{n\pi}\right]\sin n\pi x + \left[\frac{1}{n^2\pi^2} + \frac{(-1)^{n+1}}{n^2\pi^2}\right]\cos n\pi x\right\}$$