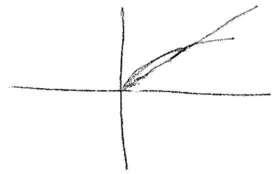
Math 2153, Quiz #13, Summer 2015 Name KEY

Instructions: Show all work. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Use Green's Theorem to evaluate $\int_C \underbrace{\cos y}_{M}\,dx + \underbrace{(xy + x\sin y)}_{N}\,dy$ on the boundary of the region between $y = x, y = \sqrt{x}$.



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \int_0^1 \int_x^{\sqrt{x}} (y + \sin y) - (-\sin y)\, dy\, dx$$

$$= \int_0^1 \int_x^{\sqrt{x}} y + 2\sin y\, dy\, dx = \int_0^1 \int_{y^2}^{y} y + 2\sin y\, dx\, dy = \int_0^1 x(y + 2\sin y)\Big|_{y^2}^{y} dy$$

$$\int_0^1 y^2 + 2y\sin y - y^3 - 2y^2 \sin y\, dy = \int_0^1 y^2 - y^3\, dy + 2\int_0^1 (y - y^2)\sin y\, dy$$

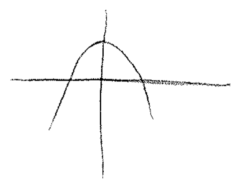
	u	dv
+	$y-y^2$	$\sin y$
−	$1-2y$	$-\cos y$
+	-2	$-\sin y$
−	0	$\cos y$

$$\frac{1}{3}y^3 - \frac{1}{4}y^4\Big|_0^1 + 2\left[-(y-y^2)\cos y + (1-2y)\sin y - 2\cos y\right|_0^1 =$$

$$\frac{1}{3} - \frac{1}{4} + 2\left[-\cancel{0\cos(1)} + (-1)\sin(1) - 2\cos(1) + \cancel{0\cos(0)} - \cancel{(-1)\sin(0)} + 2\cos(0)\right]$$

$$= \frac{1}{12} + 2\left[-\sin(1) - 2\cos 1 + 2\right] = \frac{1}{12} - 2\sin(1) - 4\cos(1) + 4$$

2. Use Green's Theorem to evaluate $\int_C \underbrace{2xy}_{M}\,dx + \underbrace{(x+y)}_{N}\,dy$ on the boundary of the region between $y = 0, y = 1 - x^2$.



$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA = \int_{-1}^1 \int_0^{1-x^2} (1 - 2x)\, dy\, dx$$

$$\int_{-1}^1 y(1-2x)\Big|_0^{1-x^2} dx = \int_{-1}^1 (1-x^2)(1-2x)\,dx = \int_{-1}^1 1 - 2x - x^2 + x^3\, dx =$$

$$\int_{-1}^1 1 - x^2\, dx + \underbrace{\cancel{\int_{-1}^1 -2x + x^3\, dx}}_{\to 0} = 2\int_0^1 1 - x^2\, dx = 2\left[x - \frac{1}{3}x^3\right|_0^1 = 2\left[1 - \frac{1}{3}\right]$$

$$= 2\left[\frac{2}{3}\right] = \frac{4}{3}$$