Math 2415, Quiz #9, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. A mass of 5 kg stretches a spring 10 centimeters. The mass is acted on by an external force of $10\sin\left(\frac{t}{2}\right)$N and moves in a medium that imparts a viscous force of 2N when the speed of the mass is 4 cm/sec. Formulate the differential equation and initial conditions to model the system if the spring starts from equilibrium.

$m = 5$ $\quad k = \frac{5 \cdot 9.8}{.1} = 490$ $\quad \gamma = \frac{2}{.4} = 5$

$$5y'' + 5y' + 490y = 10\sin\left(\tfrac{t}{2}\right)$$

2. Describe the conditions under which a spring-mass system achieves resonance.

resonance occurs when the forcing function matches the frequency of the unforced system

3. What is the difference between the frequency of a system and the quasi-frequency of the system?

the natural frequency of a system is the frequency when there is no damping. The quasi-frequency is the frequency of system under damping.

4. Determine if the vectors $\vec{x_1} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \vec{x_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \vec{x_3} = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ are linearly independent.

$$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 4 \\ 2 & 1 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

yes, they are independent

5. Find the inner product of the functions $f(t) = t^2 + 1, g(t) = -3t + 2$ under the inner product $\langle f|g\rangle = \int_0^2 f \cdot g dt$.

$$\int_0^2 (t^2+1)(-3t+2)dt = \int_0^2 -3t^3+2t^2-3t+2\,dt =$$

$$-\frac{3}{4}t^4 + \frac{2}{3}t^3 - \frac{3}{2}t^2 + 2t\Big|_0^2 =$$

$$-\frac{3}{4}(2)^4 + \frac{2}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2)$$

$$-3(4) + \frac{16}{3} - 6 + 4 = -12 - 6 + 4 + \frac{16}{3} = \frac{-26}{3}$$