Math 2415, Quiz #8, Summer 2014 Name KEY

**Instructions**: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Solve the non-homogeneous second order ODE $y'' + 2y' - 3y = -3te^{-t}$ by the method of variation of parameters.

$r^2+2r-3=0$
$(r+3)(r-1)=0$
$r=-3,1$

$W=\begin{bmatrix} e^{-3t} & e^{t} \\ -3e^{-3t} & e^{t} \end{bmatrix} = e^{-2t}+3e^{-2t} = 4e^{-2t}$

$-e^{-3t}\int \frac{e^{t}(-3te^{-t})}{4e^{-2t}}\,dt + e^{t}\int \frac{e^{-3t}\cdot(-3te^{-t})}{4e^{-2t}}\,dt$

$\frac{3}{4}e^{-3t}\int te^{2t}\,dt + \frac{3}{4}e^{t}\int te^{-2t}\,dt$

$\frac{3}{4}e^{-3t}\left[\frac{t}{2}e^{2t}-\frac{1}{4}e^{2t}\right] + \frac{3}{4}e^{t}\left[\frac{t}{-2}e^{-2t}-\frac{1}{4}e^{-2t}\right]$

$= +\frac{3}{8}te^{-t} - \frac{3}{16}e^{-t} + \frac{3}{8}te^{-t} + \frac{3}{16}e^{-t}$

$= +\frac{3}{4}te^{-t}$

$y(t) = c_1e^{-3t} + c_2e^{t} + \frac{3}{4}te^{-t}$

2. A mass weighing 8 lbs. stretches a spring 1.5 inches. The mass is also attached to a damper with coefficient $\gamma$. Determine the value of $\gamma$ for which the system is critically damped. Be sure to give the units for $\gamma$.

$\frac{8}{32} = \frac{1}{4}$ mass

$K = \frac{8}{1.5/12} = 64$

$\frac{1}{4}y'' + \gamma y' + 64y = 0$

$y'' + 4\gamma y' + 256y = 0$

$r^2 + 4\gamma r + 256 = 0$

$r = \frac{-4\gamma \pm \sqrt{16\gamma^2 - 1024}}{2}$

critically damped has discriminant $16\gamma^2 - 1024 = 0$

$\frac{16\gamma^2}{16} = \frac{1024}{16}$

$\gamma^2 = 64$

$\gamma = 8$ lbs/(ft/sec)

3. Find the inverse of the matrix $A = \begin{bmatrix} 4 & -1 \\ 2 & 8 \end{bmatrix}$.

$A^{-1} = \frac{1}{34}\begin{bmatrix} 8 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 4/17 & 1/34 \\ -1/17 & 2/17 \end{bmatrix}$