Math 2415, Quiz #7, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Find the solution to the differential equation $9y'' - 12y' + 4y = 0, y(0) = 2, y'(0) = -1$.

$9r^2 - 12r + 4 = 0$

$(3r-2)^2 = 0$

$r = 2/3$

$y = c_1 e^{2/3t} + c_2 t e^{2/3t}$ $\quad 2 = c_1$

$y' = \frac{2}{3}c_1 e^{2/3t} + c_2 e^{2/3t} + c_2(2/3)t e^{2/3t}$

$-1 = \frac{4}{3} + c_2 \quad c_2 = -7/3$

$y(t) = 2e^{2/3t} - 7/3 te^{2/3t}$

2. Use the given solution and the method of reduction of order to find the remaining solution to the differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0, y_1(t) = t, t > 0$.

$y_1' = 1$

$y_1'' = 0$

$y_2 = v \cdot y_1 = vt$

$y_2' = v'y_1 + vy_1'$

$v't + v$

$y_2'' = v''t + 2v'$

$(v''t + 2v')t^2 - t(t+2)(v't+v) + (t+2)vt = 0$

$-t^2 - 2t$

$t^3v'' + 2t^2v' - t^3v' - 2t^2v' - t^2v - 2tv + t^2v + 2tv = 0$

$t^3v'' - t^3v' = 0 \Rightarrow v'' - v' = 0 \qquad \int \frac{dv'}{dt} = \int v' \Rightarrow \int \frac{dv'}{v'} = \int dt$

$\ln v' = t \Rightarrow \int v' = \int e^t \Rightarrow v = e^t$

$y_2 = te^t$

$y = c_1 t + c_2 te^t$

3. Solve the non-homogeneous second order ODE $y'' + 2y' - 3y = -3te^{-t}$ by the method of undetermined coefficients.

$r^2+2r-3=0$

$(r+3)(r-1)=0$

$r=-3, 1$

$Y=e^{-3t}, e^{t}$

$Y_g=c_1e^{-3t}+c_2e^{t}$

$Y(t)=(At+B)e^{-t}$

$Y'(t)=Ae^{-t}+-(At+B)e^{-t} = e^{-t}(A-B-At)$

$Y''(t)=-Ae^{-t}-Ae^{-t}+(At+B)e^{-t} = e^{-t}(-2A+B+At)$

$e^{-t}(-2A+B+At)+2e^{-t}(A-B-At)-3(At+B)e^{-t} = -3te^{-t}$

$-2A+B+At+2A-2B-2At-3At-3B=-3t$

$-4A=-3$

$A=\frac{3}{4}$

$-4B=0$

$B=0$

$Y_p(t)=c_1e^{-3t}+c_2e^{t}+\frac{3}{4}te^{-t}$