Math 2415, Quiz #4, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Solve the Bernoulli equation $t^2y' + 2ty - y^3 = 0$.

$t^2y' + 2ty = y^3 \quad)y^{-3}$

$t^2y'y^{-3} + 2ty^{-2} = 1$

$z = y^{-2}$

$\frac{dz}{dt} = -2y^{-3}y'$

$-2t^2y'y^{-3} + 4ty^{-2} = -2$

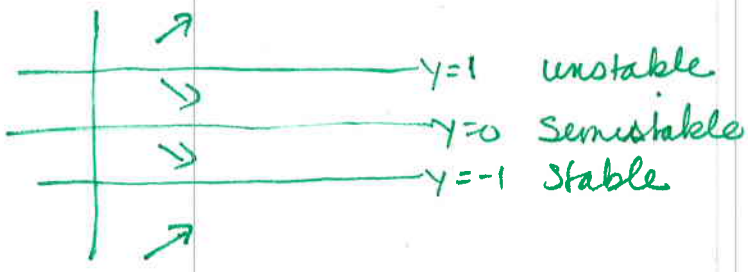
$(t^2z' - 4tz = -2) * \frac{1}{t^6}$

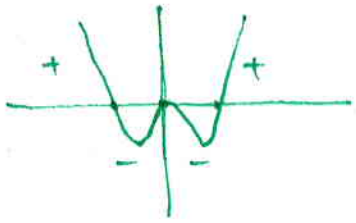
$\frac{1}{t^4}z' - \frac{4}{t^5}z = -2t^{-6} \Rightarrow \int(t^{-4}z)' = \int -2t^{-6}dt$

$(t^{-4}z = \frac{2}{5}t^{-5} + C)t^4 \Rightarrow z = \frac{2}{5t} + Ct^4 \Rightarrow \frac{1}{y^2} = \frac{2}{5t} + Ct^4$

2. Plot the direction field and phase portrait of the autonomous equation $y' = y^2(y^2 - 1)$. Label each solution as stable, semi-stable or unstable. Indicate if any of the solutions $y > 0$ qualify as thresholds or a carrying capacity.

$y^2(y^2-1) = 0 \quad y=0, y=1, y=-1$





$y=1$ is a threshold