

**Instructions:** Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Classify the following differential equations as a) linear or nonlinear, b) the order, c) ordinary or partial.

i.  $\frac{d^4 y}{dt^4} + \frac{d^2 y}{dt^2} + y = 1$

linear, 4<sup>th</sup> order, ordinary

ii.  $\frac{dy}{dt} + ty^2 = \cos(t)$

nonlinear, 1<sup>st</sup> order, ordinary

iii.  $u_{xx} + u_{yy} - u_{xy} = 0$

linear, 2<sup>nd</sup> order, partial

iv.  $\sin(t) \frac{dy}{dt} - 7ty = e^{-3t}$

linear, 1<sup>st</sup> order, ordinary

2. Use the method of integrating factors to solve the linear differential equation  $ty' - y = t^2 e^t$ .

$$y' - \frac{1}{t}y = te^t$$

$$\frac{1}{t}y' - \frac{1}{t^2}y = e^t$$

$$\int \left(\frac{1}{t}y\right)' = \int e^t$$

$$\frac{1}{t}y = e^t + C$$

$$\boxed{y = te^t + Ct}$$

$$\mu = e^{\int p(t) dt} = e^{\int \frac{1}{t} dt} = e^{-\ln t} = e^{\ln \frac{1}{t}} = \frac{1}{t}$$

3. Solve the separable differential equation  $y' = \frac{xy^3}{\sqrt{1+x^2}}$ ,  $y(0) = 1$ .

$$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}} dx$$

$$u = 1+x^2$$

$$du = 2x dx$$

$$\int \frac{1}{2} u^{-1/2} du$$

$$\int y^{-3} dy =$$

$$\frac{y^{-2}}{-2} = \frac{1}{2} \sqrt{u} + C$$

$$-\frac{1}{2y^2} = \sqrt{1+x^2} + C$$

$$y^2 = -\frac{1}{2\sqrt{1+x^2} + C}$$