Math 2415, Quiz #15, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Solve the given initial value problem $\vec{x}' = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}\vec{x}, \vec{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

$(5-\lambda)(1-\lambda)+3 = 5-6\lambda+\lambda^2+3 = \lambda^2-6\lambda+8 = 0 \quad (\lambda-4)(\lambda-2)=0$

$\lambda = 2, \lambda = 4$

$\lambda=2$: $\begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$ $3x_1 = x_2$ $\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\lambda=4$: $\begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$ $x_1 = x_2$ $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\vec{x} = c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix}e^{2t} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{4t} \qquad \begin{bmatrix} 2 \\ -1 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 3 & 1 & -1 \end{array}\right] \Rightarrow \begin{array}{l} c_1 = -3/2 \\ c_2 = 7/2 \end{array}$

$\vec{x} = -\frac{3}{2}\begin{bmatrix} 1 \\ 3 \end{bmatrix}e^{2t} + \frac{7}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{4t}$

2. Solve the given initial value problem $\vec{x}' = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}\vec{x}, \vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

$(1-\lambda)(-3-\lambda)+5 = -3+3\lambda-\lambda+\lambda^2+5 = \lambda^2+2\lambda+2 = 0$

$\lambda = \frac{-2 \pm \sqrt{4-4(2)}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$

$\begin{matrix} 1-(-1\pm i) & -5 \\ 1 & -3-(-1\pm i) \end{matrix}$

$-3-(-1\pm i)$

$-3+1 \mp i$

$-2 \mp i$

$x_1 = (2 \pm i)x_2$ $\vec{v}_{1,2} = \begin{bmatrix} 2+i \\ 1 \end{bmatrix}$

$\vec{x} = e^{-t}(\cos t + i\sin t)\begin{pmatrix} 2+i \\ 1 \end{pmatrix} = e^{-t}\begin{pmatrix} 2\cos t + 2i\sin t + i\cos t - \sin t \\ \cos t + i\sin t \end{pmatrix} =$

$c_1 e^{-t}\begin{pmatrix} 2\cos t - \sin t \\ \cos t \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 2\sin t + \cos t \\ \sin t \end{pmatrix} = e^{-t}\begin{bmatrix} \cos t - 3\sin t \\ \cos t - \sin t \end{bmatrix}$

$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 0 & 1 \end{array}\right] \Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$