Math 2415, Quiz #14, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Determine if the vectors $\vec{x_1} = \begin{bmatrix} 2\sin t \\ \sin t \end{bmatrix}, \vec{x_2} = \begin{bmatrix} e^t \\ t\sin t \end{bmatrix}$ are linearly independent. [Hint: compute the Wronskian.]

$\begin{bmatrix} 2\sin t & e^t \\ \sin t & t\sin t \end{bmatrix} = 2t\sin t - e^t \sin t \neq 0$ unless $t=0$

they are independent

2. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix}$. If the eigenvectors are real, sketch them on a graph and determine the long-run behaviour of the system near those eigenvectors by plotting sample trajectories. If the eigenvectors are imaginary, you don't need to plot them, but you do need to determine the general long-run behaviour.

$(1-\lambda)(3-\lambda)-8=0$

$\lambda^2-4\lambda+3-8=0$

$\lambda^2-4\lambda-5=0$

$(\lambda-5)(\lambda+1)=0$

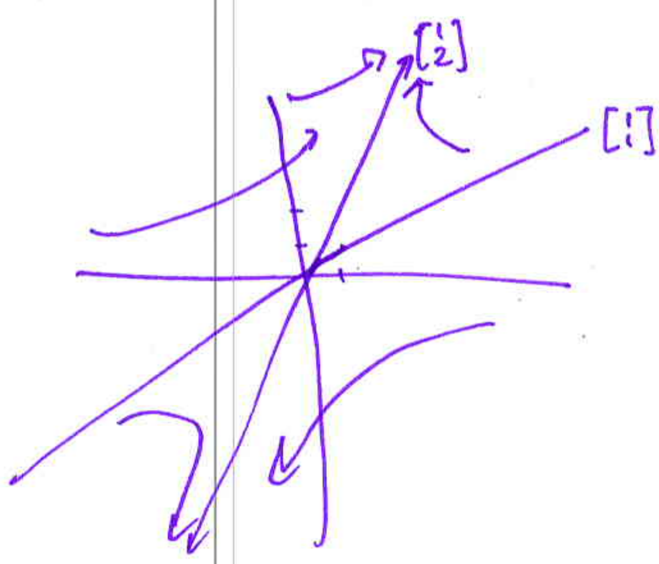
$\lambda=5, \lambda=-1$

$\lambda=5$

$\begin{bmatrix} -4 & -2 \\ -4 & -2 \end{bmatrix}$ $\quad \frac{4x_1}{2}=\frac{2x_2}{2}$ $\quad 2x_1=x_2$ $\quad \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda=-1$

$\begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix}$ $\quad 2x_1=2x_2$ $\quad x_1=x_2$ $\quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



trajectory approaches vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

trajectories on vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ oscillate back and forth but stay same distance from origin