Math 2366, Quiz #9, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. What is the probability that a five-card poker hand contains two pairs (that is two of a kind, a second two of a kind, and one other card that does not match the other two)?

$$\frac{(13)\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}} \approx .007923$$

2. Find the probability of winning a lottery's largest prize (all six numbers correct), if there are 56 choices for the first selection.

$$\frac{1}{\binom{56}{6}} = 3.08 \times 10^{-8}$$

3. What is the probability that a fair coin is flipped 10 times and results in six heads?

$$\binom{10}{6}(1/2)^6(1/2)^4 = .205$$

4. What is the probability that a fair coin is flipped 20 times and there are fewer than 4 heads?

$$\binom{20}{0}(1/2)^{10} + \binom{20}{1}(1/2)^{10} + \binom{20}{2}(1/2)^{10} + \binom{20}{3}(1/2)^{10} \approx .001288$$

5. Suppose that you have a \$1 cost to buy a lottery ticket, and that if you get 6 numbers correct from 50 possibilities you could win \$10,000,000 (otherwise, you lose everything). What is the expected value?

$$-\$1 + \$10{,}000{,}000\frac{1}{\binom{50}{6}} = -.3707$$

6. Suppose that you roll a pair of dice until you get snake eyes (two ones). What is the expected number of rolls that will be needed?

$p(1,1) = 1/36$

$E(x) = 1/p = 36$

7. Based on the table below, determine if the two variables are independent. If they are, explain how you know. If they are not, explain why not.

$p(x,y)$	0	1	2	3	
0	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{20}$	$\frac{1}{40}$	$3/10$
1	$\frac{1}{16}$	$\frac{1}{25}$	$\frac{1}{8}$	$\frac{1}{16}$	$29/100$
2	$\frac{1}{50}$	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{1}{100}$	$23/100$
3	$\frac{1}{16}$	$\frac{1}{75}$	$\frac{1}{15}$	$\frac{3}{80}$	$9/50$
	.245 $49/200$	$\frac{131}{600}$	$\frac{241}{600}$	$\frac{27}{200}$	

$E(xy) = 1(1/25) + 2(1/8) + 3(1/16) + 2(1/25) + 4(4/25) + 6(\frac{1}{100}) +$
$3(1/75) + 6(1/15) + 9(3/80) = \frac{407}{200} \approx 2.035$

$E(x) = \frac{131}{600} + 2(\frac{241}{600}) + 3(\frac{27}{200}) = 107/75 \approx 1.4267$

$E(y) = \frac{29}{100} + 2(\frac{23}{100}) + 3(\frac{9}{50}) = 129/100 = 1.29$

$E(X)E(Y) = \frac{107}{75} \cdot \frac{129}{100} = \frac{4061}{2500} = 1.8404$

$E(XY) \neq E(X)E(Y)$ $\therefore$ not independent