Math 2366, Quiz #2, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. For the argument below, state the premises, the conclusion and any rules of inference used at each step.

   Someone in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.

1 $\exists x\, W(x)$ premise
2 $\forall x\, (W(x) \rightarrow O(x))$ premise
3 $W(c)$ existential instantiation (1)
4 $O(c)$ from 2 modus ponens
5 $\exists x\, O(x)$ from 4 existential generalization

2. Determine if the following argument is valid. If it is, explain the rules of inference used. If it is find, find the error and the correct conclusion.

   If n is a real number with n>2, then $n^2 > 4$. *Suppose that* $n \leq 2$. *Then* $n^2 \leq 4$.

it is not valid. Shown w/ counter example: $n = -3$, satisfies $n \leq 2$, but n^2 is not ≤ 4.

this is an inverse relationship, not the contrapositive.

3. Prove that if mn is even, then either m is even or n is even. Assume that mn, m, n are all integers.

proof by cases: ① both m & n are odd, ② m is even, ③ n is even

① Suppose both m & n are odd, ie $m = 2a+1$, $n = 2b+1$ for some $a, b \in \mathbb{Z}$
then $mn = (2a+1)(2b+1) = 4ba + 2a + 2b + 1 = 2(ba+a+b)+1$ which is odd
Since mn is not even, exclude this case. m & n cannot both be odd.

② Suppose m is even & n is anything: ie. $m = 2k$ & $n = n$ for $k \in \mathbb{Z}$
then $mn = 2k(n) = 2(kn)$ which is even.

③ Suppose n is even & m is anything: ie. $n = 2k$, & $m = m$.
the $mn = m2k = 2(km)$ which is even.

therefore, the only way for mn to be even is for m to be even or n to be even.

4. Prove that there are no positive integers such that $n^2 + n^3 = 100$.

Proof by exhaustion:

$n=1$ $1^2+1^3 = 2 \neq 100$

$n=2$ $2^2+2^3 = 4+8 = 12 \neq 100$

$n=3$ $3^2+3^3 = 9+27 = 36 \neq 100$

$n=4$ $4^2+4^3 = 16+64 = 80 \neq 100$

$n=5$ $5^2+5^3 = 25+125 = 150 \neq 100$

all other integers >0 will have larger sums and none are equal to 100.

5. Prove that $\sqrt[3]{2}$ is irrational. [Hint: the proof is similar to the one for showing that the $\sqrt{2}$ is irrational.]

Suppose that $\sqrt[3]{2}$ is rational; i.e. $\sqrt[3]{3} = \frac{b}{a}$ for some integers $a, b, a \neq 0$ and $\frac{b}{a}$ is reduced form (i.e. they have no common factors)

then $\left(\sqrt[3]{2} = \frac{b}{a}\right)^3 \Rightarrow 2 = \frac{b^3}{a^3} \Rightarrow 2a^3 = b^3$

if a is an integer then $2a^3$ is even, and so b^3 must be even, but for b^3 to be even then b must be even $\Rightarrow b = 2c \Rightarrow$ $2a^3 = (2c)^3 \Rightarrow$

$2a^3 = 8c^3 \Rightarrow$

$a^3 = 4c^3$

but this implies that a must be even by the same reasoning. However, since we said $\frac{b}{a}$ has no common factors, this is impossible (if a & b are both even, the common factor is 2), therefore $\sqrt[3]{2}$ is irrational.