2366 Homework #1 Key

1.a. I did not buy a lottery ticket.

b. I either bought a lottery ticket or I won the million dollar jackpot.

c. If I bought a lottery ticket, then I won the million dollar jackpot.

d. I bought a lottery ticket and I won the million dollar jackpot.

e. I bought a lottery ticket if and only if I won the million dollar jackpot.

f. That I did not buy a lottery ticket means that I did not win the million dollar jackpot.

g. I did not ~~b~~ buy a lottery ticket and I did not win the million dollar jackpot.

h. I did not buy a lottery ticket or I both bought a ticket and won the million dollar jackpot.

2.a. $\neg p$

b. $p \wedge \neg q$

c. $p \rightarrow q$

d. $\neg p \rightarrow \neg q$

e. $p \rightarrow q$

f. $\neg p \wedge q$

g. $q \rightarrow p$

3. a. $2^1 = 2$

b. $2^4 = 16$

c. $2^6 = 64$

d. $2^4 = 16$

4. a.

p	$\neg p$
T	F
F	T

b.

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

3.c.

p	q	$p \oplus q$	$\neg q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow \neg q) \land (p \oplus q)$
T	T	F	F	F	F
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	F	F

3d.

p	q	r	$p \lor q$	$\neg r$	$(p \lor q) \land \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

e.

p	q	r	s	$(p \land q)$	$r \to s$	$(p \land q) \oplus (r \to s)$
T	T	T	T	T	T	F
T	T	T	F	T	F	T
T	T	F	T	T	T	F
T	T	F	F	T	T	F
T	F	T	T	F	T	T
T	F	T	F	F	F	F
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	T	F	F	F	F
F	T	F	T	F	T	T
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	T	F	F	F	F
F	F	F	T	F	T	T
F	F	F	F	F	T	T

5. a. Suppose A is telling the truth. if so, Then C's claim that he is the spy must be false, so he is not the spy. If B is correct that A is the knight, then this is consistent. However, That would make B the spy. $\boxed{A=\text{knight},\ B=\text{spy},\ C=\text{knave.}}$ So There is at least one solution. Suppose that C is telling the truth. Then A is lying so he can't be the knight. Also B can't be the knight since saying that A is a knight would have to be a lie. If A is lying then he can't be the knight, but this means C would have to be then and That is also impossible

b. no consistent solution. If A is telling the truth, then he must be the knight & one of The other two must be lying. But anyone who always lies and says "I am the knave" is self-contradictory.

c. Suppose A is the knight. Then what B says is true, so he must be the spy, but then This is consistent w/ C – only C must be The knave and so he must be lying, so this is not possible. Therefore, we must conclude that A is lying. He could be the spy. Then what B says is true, so that could be the knight, but Then C would also be telling the truth, so that won't work. Suppose That A is the knave. Then B must be lying, so he must be the spy, and Then what C says is true so he could be the knight.

$A=\text{knave},\ B=\text{spy},\ C=\text{knight}$ at least one solution.

d. Suppose A is telling the truth. then he must be the knight since saying you are not the spy when you are would not be truthful. Then B could be the spy, but Then C would have to be The knave saying he is not The spy, but that would be the truth, So That is not consistent. There is no solution here since all roads lead to The same problem.

6. a. $(A \wedge B) \wedge \neg(B \wedge C) = X$

$\neg(C \vee D) \vee \neg(B \wedge C) = Y$

b. $\neg(A \vee B) \wedge \neg C \wedge (C \vee D)$

6c. $(\neg X \wedge Y) \vee (X \wedge \neg Y)$

d. $\neg(A \vee B) \vee (B \wedge C) = Q$

$\neg\big(Y(\text{James}) \wedge S(\text{James})\big) = \neg Y(\text{James}) \vee \neg S(\text{James})$

either James is not young or James is not strong.

8a.

p	q	$(p \wedge q)$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

← this is not a tautology

b.

p	q	r	$p \to r$	$q \to r$	$(p \to q) \wedge (q \to r)$	$(p \vee q)$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

These two columns are identical So identical inputs yield identical outputs so they are logically equivalent.

c.

p	$\neg p$	$p \vee \neg p$	$\neg(p \vee \neg p)$
T	F	T	F
F	T	T	F

← by definition a statement which is always false is unsatisfiable

↑ tautology

9a. $p \downarrow q \equiv \neg(p \wedge q)$

p	q	$p \wedge q$	$p \downarrow q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

b. $p \uparrow q \equiv \neg(p \vee q)$

p	q	$p \vee q$	$p \uparrow q$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

10 a. true b. false c. false d. false e. true f. false

11 a. $D(x)$ = x is studying discrete math.

$\forall x D(x)$ true on domain of students in 2366.
false on domain of all CSCC students

b. $O(x)$ = x is older than 21 years.

$\exists x O(x)$ true of CSCC students
false for students in kindergarten

c. $S(x,y)$ x has the same name as y.

$\forall x \exists! y \, (x \neq y) \, S(x,y)$

generally false. to be true, the domain would have to be selected very carefully

d. $L(x)$ = x has taken a course in logic programming.

$\forall x \neg L(x)$

true on domain of an English class (probably)
false on a domain of computer programming

e. $P(x)$ = x is perfect

$\forall x \neg P(x)$ true on every domain.
Is there such a thing as perfect? is it possible for someone to be perfect? If so, maybe it can be false, but I don't think so. :)

f. $C(x)$ = x is in the correct place

$\exists x \neg C(x)$ true domain of books in library before inventory
possibly false afterwards

11g. $C(x)$ = x is ~~on~~ in the correct place. $E(x)$ x is in excellent condition.

$\exists x\,(\neg C(x) \land E(x))$

12 a. Someone has sent an email to someone in the class.

b. Everyone has sent an email to someone.

c. There is someone to whom everyone has sent an email.

d. Everyone has sent an email to everyone in the class.

13. a. $\forall x\ L(x, \text{Jerry})$

b. $\forall x\ \exists y\ L(x, y)$

c. $\exists y\ \neg L(\text{Lydia}, y)$

d. $\forall x\ \forall y\ \neg L(x, y)$

e. $\exists! x\ L(\text{Lynn}, x) \land \exists! y\ (x \neq y)\ L(\text{Lynn}, y)$

f. $\exists! y\ \forall x\ L(x, y)$

g. $\exists x\ \forall y\ \neg L(y, x)$.

14. a. $Q(1,1) \Rightarrow 1+1 = 1-1$ false $2 \neq 0$

b. $\forall y\ Q(1,y) \Rightarrow 1+y = 1-y \Rightarrow y=-y$ false $0=0$ but not ally $1 \neq -1$

c. $\exists x \exists y\ Q(x,y) \Rightarrow x+y = x-y \Rightarrow y=-y$ true

d. $\forall n\ \exists m\ (n^2 < m)$ true

e. $\exists n\ \forall m\ (nm = m)$ true

f. $\forall n\ \forall m\ \exists p\ (p = (m+n)/2)$ false $\frac{\overset{m}{3}+\overset{n}{2}}{2}$ is not an integer

g. $\forall x \forall y\ (x^2 = y^2 \Rightarrow x = y)$ false $(1^2) = (-1)^2$ but $1 \neq -1$

15 a. $\exists y\ \exists x\ (\neg P(x,y) \land \neg Q(x,y))$

b. $\exists x\ (\forall y\ \exists z\ \neg P(x,y,z) \lor \forall z\ \exists y\ \neg P(x,y,z))$