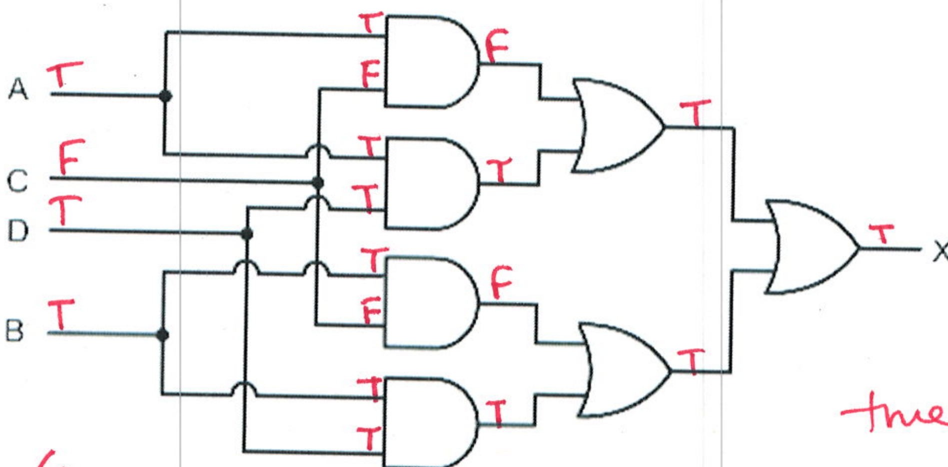
Math 2366, Exam #1, Summer 2014 Name KEY

Instructions: Show all work. Justify answers as completely as possible. If you are asked to prove something, mere computation is not enough. You must explain your reasoning. Be sure to state your conclusion clearly. Incomplete work or justification will not receive full credit. Use exact answers unless specifically asked to round.

1. Complete the truth table below for the statement $(p\oplus q)\wedge(q\Rightarrow\neg p)$. (10 points)

p	q	$\neg p$	$p\oplus q$	$q\Rightarrow\neg p$	$(p\oplus q)\wedge(q\Rightarrow\neg p)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	T	F

2. For the logic gate shown below, write the equivalent expression in propositional logic. Then is we assume the inputs to A,B and D are true, and C is false, what is the output of the gate? (10 points)



true

$((A\wedge C)\vee(A\wedge D))\vee((C\wedge B)\vee(D\wedge B))$

3. Determine whether $(\neg p\wedge(p\Rightarrow q))\Rightarrow\neg q$ is a tautology, unsatisfiable or neither. (5 points)

p	q	$\neg p$	$p\to q$	$\neg p\wedge(p\to q)$	$\neg q$	$(\neg p\wedge(p\to q))\to\neg q$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

neither

4. Determine the truth value of the statements below if the domain if each variable is the set of integers. (3 points each)
   a. $\forall n(n^2 \geq 0)$ T

   b. $\exists n(n^2 = 2)$ F

   c. $\forall n(n^2 \geq n)$ T

   d. $\exists n(n = -n)$ T

5. Translate the sentence 'Someone in your class can speak Hindi' into predicates and logical connectives. Be sure to define each predicate you use, and state the domain of any variables. (6 points)

H(x) = 'x speaks Hindi'.

$\exists x\, H(x)$

$x \in$ {those in your class}

6. Describe the difference between $\forall x \exists y(Px, y)$ and $\exists y \forall x P(x, y)$. Relate a scenario where the difference is important (using the real world or numbers). (8 points)

Suppose P(x, y) is "x ate y"

$\forall x \exists y\, P(x,y) \Rightarrow$ '~~for~~ every possible person x, they ate something y'

but

$\exists y \forall x\, P(x,y) \Rightarrow$ 'for some y, every possible x ate that.'

7. Identify the error(s) in the following argument that claims to show that if $\exists x P(x) \wedge \exists x Q(x)$ is true, then $\exists x(P(x) \wedge Q(x))$ is true. (6 points)

Step #	Step	Reason(s)
1.	$\exists x P(x) \wedge \exists x Q(x)$	Premise
2.	$\exists x P(x)$	Simplification from (1)
3.	$P(c)$	Existential instantiation from (2)
4.	$\exists x Q(x)$	Simplification from (1)
5.	$Q(c)$	Existential instantiation from (4) ← use a diff. variable from any used previously
6.	$P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7.	$\exists x(P(x) \wedge Q(x))$	Existential generalization from (6)

error – c's need not be the same

8. Prove that if $3n + 2$ is even, then n is even. (10 points)

Suppose that $3n+2 = 2k$ (even)

then $3n = 2k-2 = 2(k-1)$ which is also even.

but the only way $3n$ can be even is

if n is even.

QED.

9. Show that if n is an odd integer, then there is a unique integer k such that n is the sum of $k-2$ and $k+3$. (8 points)

$n = 2k+1$ (n is odd) for some unique integer k

but $2k+1 = k+k+3-2 = (k+3)+(k-2)$

for some integer k.

QED.

10. Find the $\mathcal{P}(A)$ if $A = \{m, n, o, p\}$. (6 points)

$\{\emptyset, \{m\}, \{n\}, \{o\}, \{p\}, \{m,n\}, \{m,o\}, \{m,p\}, \{n,o\},$
$\{n,p\}, \{o,p\}, \{m,n,o\}, \{m,n,p\}, \{n,o,p\},$
$\{m,n,o,p\}\}$

11. If $A_n = \left\{\frac{i}{n}\right\}_{i=1}^{n}$ for some integer, find $\bigcup_{n=1}^{4} A_n$. (8 points)

$A_1 = \{1\}$, $A_2 = \{\frac{1}{2}, \frac{2}{2}\}$, $A_3 = \{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}\}$

$A_4 = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{4}{4}\}$

$$\bigcup_{n=1}^{4} A_n = \{1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{2}{3}, \tfrac{1}{4}, \tfrac{3}{4}\}$$

12. Give an explicit formula for a function from $\mathbb{R}$ to $\mathbb{R}$ that is: (4 points each)

answers will vary

a. One-to-one but not onto.

$f(x) = \tan^{-1}(x)$

b. Onto, but not one-to-one.

$f(x) = \ln(x^2)$ for $x \neq 0$, $\{1\}$ for $x = 0$

c. One-to-one and onto

$y = x$

d. Neither one-to-one, nor onto.

$f(x) = \sqrt{x}$

13. Find $f(S)$ if $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$ and $S = \{1, 5, 7, 11\}$. (5 points)

$f(1) = \lfloor \frac{1}{3} \rfloor = 0$

$f(5) = \lfloor \frac{25}{3} \rfloor = 8$

$f(7) = \lfloor \frac{49}{3} \rfloor = 16$

$f(11) = \lfloor \frac{121}{3} \rfloor = 40$

$f(S) = \{0, 8, 1, 4\}$

14. Find at least three different sequences whose first three terms are 1, 2, 4... and which are generated by a simple rule. List at least three more terms in each sequence, and write the rule for each. (9 points)

$\{1, 2, 4, 8, 16, 32, \ldots\} \Rightarrow a_n = \{2^n\}_{n=0}$

$\{1, 2, 4, 21, 116, 715 \ldots\} \Rightarrow a_n = \{(n+1)! - n\}_{n=0}$

$\{1, 2, 4, 7, 11, 16, \ldots\} \Rightarrow a_n = \{a_{n-1} + n\}, a_0 = 1$

your answers may vary

15. Determine whether each of the sets below is countable or uncountable (or finite). (3 points each)
    a. Integers not divisible by 3

       Countable

    b. The real numbers with decimal representations of all 1s.

       Countable

    c. All positive rational numbers that cannot be written with a denominator less than 4.

       Countable

    d. The real numbers between 0 and 2.

       uncountable

16. Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$. You may not depend on a Venn diagram as your only 'proof'. Use elements of the set. (12 points)

Let $x \in \overline{A \cap B}$. Then x is either not in A or not in B so that its not in the intersection.

if x is not in A then its in $\bar{A}$. Then x is in $\bar{A} \cup \bar{B}$ by definition of the union. if x is not in B, Then x is in $\bar{B}$, then x is in $\bar{A} \cup \bar{B}$ by definition of union.

$\therefore \overline{A \cap B} \subset \bar{A} \cup \bar{B}$.

On the other hand, if x is in $\bar{A} \cup \bar{B}$, Then either x is in $\bar{A}$ or $x \in \bar{B}$.
if $x \in \bar{A}$, Then x is not in A so x is not in $A \cap B \Rightarrow x \in \overline{A \cap B}$.
if $x \in \bar{B}$, then x is not in B so x is not in $A \cap B$, but this means that x is in $\overline{A \cap B}$. Therefore $\bar{A} \cup \bar{B} \subset \overline{A \cap B}$.
Since The sets are subsets of each other, then they must be equal.
$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$.