Math 1030, Exam #2, Summer 2014 Name KEY

**Instructions**: Show all work. Use exact answers unless specifically asked to round. Reduce as much as possible. Be sure to answer all parts of each question. Be sure to declare your variables in word problems and state units in your final answer.

1. The sum of two numbers is 127. Twice the smaller number subtracted from the larger number is 4. Find the two numbers. [You must show work to earn full credit on this problem. Trial and error that yields a guess without the algebra will earn only 1 point.] (10 points)

Let x be the smaller #
& y be the larger one

$x + y = 127$

$y - 2x = 4$

$y = 2x + 4$

$x + (2x+4) = 127$

$3x + 4 = 127$

$\frac{3x}{3} = \frac{123}{3}$

$x = 41$

$y = 41(2) + 4 = 86$

the two numbers are 41 & 86

2. You own a nut company that sells peanuts for \$4.50/pound and cashews for \$6.00/pound. You'd like to make a mid-priced mixed-nuts style that is part peanuts and part cashews. You'd like to make up 75 pounds of the peanut-cashew mixture and you want to sell it for \$5.40/pound. Find the amount of peanuts and cashews needed for the mixture. (10 points)

x = Qty of peanuts in pounds
y = Qty of cashews in pounds

$4.5x + 6y = 5.4(75) = 405$

$x + y = 75 \rightarrow y = 75 - x \Rightarrow y = 75 - 30 = 45$

$4.5x + 6(75 - x) = 405$

$4.5x + 450 - 6x = 405$

$\frac{-1.5x}{-1.5} = \frac{-45}{-1.5}$

$x = 30$

you need 30 lbs of peanuts and 45 lbs of cashews

3. Suppose that in travelling down a river 60 miles in a small boat, it takes you 6 hours to make the journey, but that travelling back to your starting point on the same river it takes 15 hours. Find the speed of the boat in still water and the speed of the current. (10 points)

$d = rt \Rightarrow r = \frac{d}{t}$

$r = b + c$

$b$ = boat speed + current speed (downstream) $c$ =

$r = b - c$ (upstream)

$\frac{60}{b} = 10 = b + c$

$\frac{60}{15} = 4 = b - c$

$14 = 2b \Rightarrow b = 7$

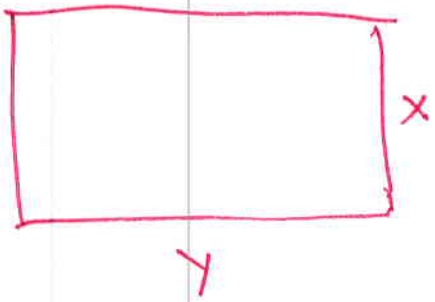
$10 = b + c$

$10 = 7 + c$

$3 = c$

The boat speed is 7 mph & the current speed is 3 mph.

4. A gardener has a field she intends to plant with flowers. The field is rectangular. The length of the field is nine feet more than five times the width. The perimeter of the field is 198 feet. Draw a picture and label it appropriately. Find the dimensions of the field. (10 points)




$y = 5x + 9$

$P = 2x + 2y = 198$

$2x + 2(5x + 9) = 198$

$2x + 10x + 18 = 198$

$\frac{12x}{12} = \frac{180}{12}$

$x = 15$

$y = 5(15) + 9 = 84$

the field is 15 feet by 84 feet.

5. Graph the system of inequalities on the attached graphs. Be sure to clearly shade or otherwise indicate the solution region. (10 points each)

a. $\begin{cases} 3x - 2y \leq 12 \\ 4x + y \leq 4 \end{cases}$

$y \leq -4x + 4$

Check origin

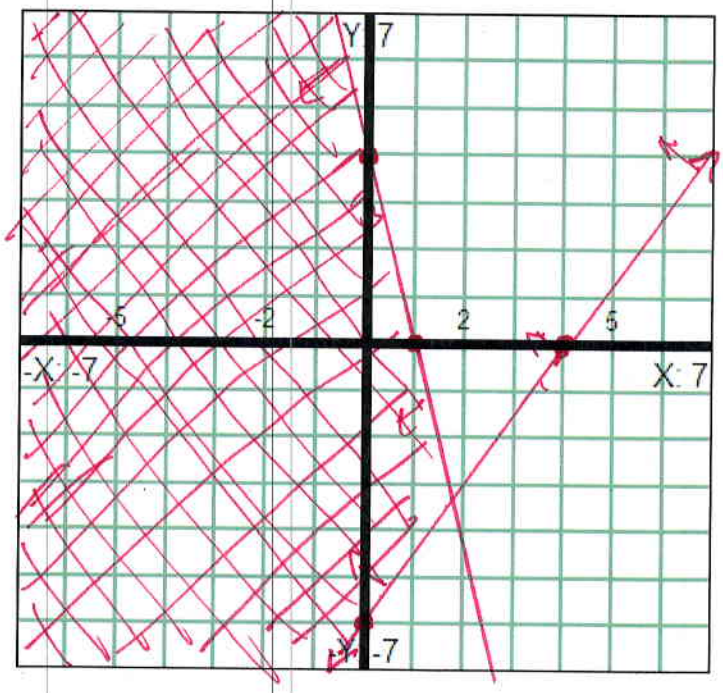
(0,0)

$0 - 0 \leq 12$

true

$0 + 0 \leq 4$

true



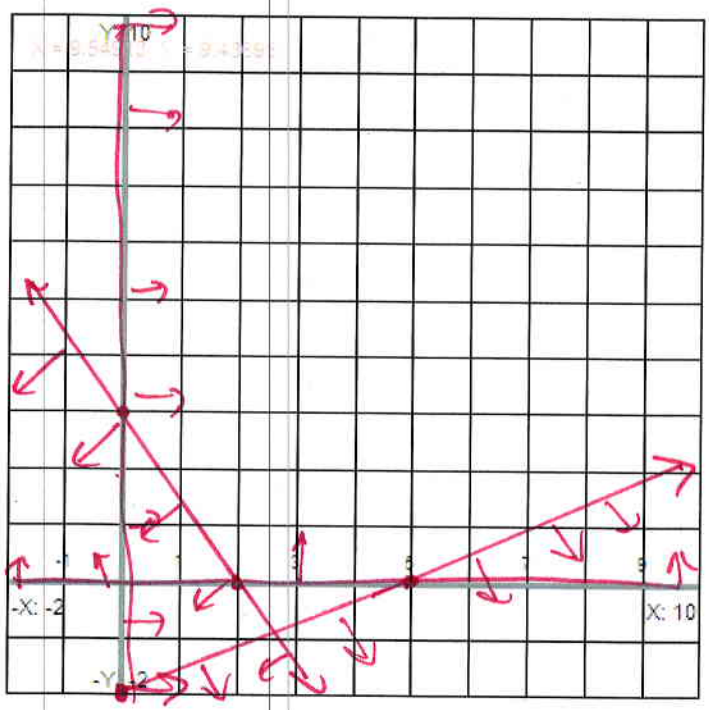

b. $\begin{cases} 2x - 5y \geq 10 \\ 3x + 2y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$

$\frac{-5y}{-5} \geq \frac{-2x}{-5} + \frac{10}{-5}$

$y \leq \frac{2}{5}x - 2$

$\frac{2y}{2} \leq \frac{-3x + 6}{2}$

$y \leq -\frac{3}{2}x + 3$




Check origin

(0,0) $0 \geq 10$ false

$0 \leq 6$ true

No region satisfies all the conditions $\Rightarrow$ no solution

6. State the degree of the polynomial. Also state whether the polynomial is a monomial, a binomial or a trinomial, or none of these. (4 points each)

a. $-7x^3 - x^5 + 8$ degree 5, trinomial

b. $x^4y^2 + 16y^3$ degree 6, binomial

c. $(-2)^3$ degree 0, monomial

d. $5p^2 + q^2 + 2pq - 1$ degree 2, none of these

7. Give an example of an expression that is NOT a polynomial and one that is. How would you explain the difference? (5 points)

not a polynomial $\frac{1}{x}$, $2^x$, $\sqrt{x}$, etc

polynomial $x^2$, $x^4$, 6, etc.

a polynomial contains a variable(s) raised to a whole # power (0, 1, 2, 3, ...)

8. Simplify the expressions and combine like terms.

a. $(3x^2 + 5x - 16) - 2(x^3 - 6x - 9)$ (5 points)

$3x^2 + 5x - 16 - 2x^3 + 12x + 18$

$-2x^3 + 3x^2 + 17x + 2$

b. $\left(\frac{-2wz^3}{5w^4z}\right)^2$ (7 points)

$\frac{4w^2z^6}{25w^8z^2} = \frac{4z^4}{25w^6}$

c. $3t^4(5t^2 - 3t + 7)$ (5 points)

$15t^6 - 9t^5 + 21t^4$

d. $(3x^0 - 7^0)^2$ (5 points)

$[3(1) - 1]^2 = [3 - 1]^2 = [2]^2 = 4$

e. $(2x - 5)(9x + 7)$ (6 points)

$18x^2 + 14x - 45x - 35$

$18x^2 - 31x - 35$

f. $\left(\frac{-2a^2b^6}{3a^{-3}b}\right)^{-3}\left(\frac{ab^5}{9a^{-5}b^4}\right)$ (9 points)

$\frac{(-2)^{-3}a^{-6}b^{-18}}{3^{-3}a^9b^{-3}} \cdot \frac{ab^5}{9a^{-5}b^4} = \frac{\frac{3}{27}b^3}{8a^9a^6b^{18}} \cdot \frac{a \cdot a^5b^5}{9b^4} =$

$-\frac{3a^6b^8}{8a^{15}b^{22}} = \boxed{\frac{-3}{8a^9b^{14}}}$

g. $(b - 4c)^2$ (5 points)

$b^2 - 8bc + 16c^2$

h. $(r+5)(r+2)(r-2)$ (7 points)

$(r+5)(r^2-4)$

$r^3+5r^2-4r-20$

i. $(n-2)(n^2+2n+4)$ (7 points)

$n^3+2n^2+4n-2n^2-4n-8$

$=n^3-8$

9. Give an example that demonstrates why any number raised to the zero power is equal to one (i.e. why is $a^0=1$). [Hint: use other properties of exponents.] (6 points)

$\frac{2^3}{2^3}=2^{3-3}=2^0$

but $\frac{2^3}{2^3}=\frac{8}{8}=1$ so $2^0$ must also equal 1

the same for any number except possibly zero