Math 2568, Quiz #8, Summer 2013 Name KEY

Instructions: You may use the calculator to perform row operations, but other work should be shown. Where no work beyond that is required, justify your answer with an explanation. Use exact values.

1. Find a basis for the space spanned by the vectors $\begin{bmatrix}2\\6\\0\\1\end{bmatrix}, \begin{bmatrix}1\\3\\5\\2\end{bmatrix}, \begin{bmatrix}1\\3\\-5\\-1\end{bmatrix}, \begin{bmatrix}5\\2\\1\\0\end{bmatrix}, \begin{bmatrix}3\\0\\2\\3\end{bmatrix}$. What is the dimension of the space.

rref $\rightarrow \begin{bmatrix}1&0&1&0&0\\0&1&-1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$

↑ dependent

$\text{Span}\left\{\begin{bmatrix}2\\6\\0\\1\end{bmatrix}, \begin{bmatrix}1\\3\\5\\2\end{bmatrix}, \begin{bmatrix}5\\2\\1\\0\end{bmatrix}, \begin{bmatrix}3\\0\\2\\3\end{bmatrix}\right\}$

this is $\mathbb{R}^4$ So dim = 4

2. For the polynomial $p(t) = 4 + 5t - 2t^2 + 7t^4$, find an expression for the polynomial in terms of the basis $\{t, 1-2t, 2t+t^2, t-3t^3, 1+t^4\}$. If there is no expression (or no unique expression), explain why.

$P_B \begin{bmatrix}0&1&0&0&1\\1&-2&2&1&0\\0&0&1&0&0\\0&0&0&-3&0\\0&0&0&0&1\end{bmatrix} \quad \vec{x} = \begin{bmatrix}4\\5\\2\\0\\7\end{bmatrix}$

$P_B^{-1}\vec{x} = \begin{bmatrix}-5\\-3\\2\\0\\7\end{bmatrix} = [x]_B$

unique since B is a basis