Math 2568, Quiz #10, Summer 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. All answers must be justified with work or written explanation.

1. Given the basis $\mathcal{B}=\left\{\begin{bmatrix}1\\1\\3\end{bmatrix},\begin{bmatrix}0\\4\\-2\end{bmatrix},\begin{bmatrix}1\\7\\0\end{bmatrix}\right\}$ and $\mathcal{C}=\left\{\begin{bmatrix}2\\-1\\0\end{bmatrix},\begin{bmatrix}1\\5\\-3\end{bmatrix},\begin{bmatrix}3\\0\\2\end{bmatrix}\right\}$, suppose that $[\vec{x}]_B=\begin{bmatrix}9\\-5\\3\end{bmatrix}$. Find $\vec{x}$ in the standard basis and $[\vec{x}]_C$. As part of your work, you should give any matrices used in your calculations.

$P_B=\begin{bmatrix}1&0&1\\1&4&7\\3&-2&0\end{bmatrix}$ $P_C=\begin{bmatrix}2&1&3\\-1&5&0\\0&-3&2\end{bmatrix}$

$P_{C\leftarrow B}=P_C^{-1}P_B=\begin{bmatrix}-46/31&-14/31&-67/31\\-3/31&22/31&30/31\\42/31&2/31&45/31\end{bmatrix}$

$\vec{x}=\begin{bmatrix}12\\10\\37\end{bmatrix}$ $[\vec{x}]_C=\begin{bmatrix}-545/31\\-47/31\\503/31\end{bmatrix}$

2. 100 mice are set to run a maze that has three feeding locations in it. The mice are divided up and split among the various locations with 25% starting at Location A, 35% starting at Location B, and 40% starting at Location C. If a mouse is at location A, it has a 75% chance of returning to the same location, and a 10% chance of ending up at Location B. If a mouse is at location B, it has a 50% chance of returning to Location B, and a 30% chance of ending up at Location C. And if the mouse starts at Location C, it has a 40% chance of going back to Location C, and a 30% chance of ending up at Location A. Set up the stochastic matrix representing the Markov chain and the initial state vector. Use this information to algebraically determine the equilibrium vector (use the back of the page). After the mice have had a chance to run around the maze for a while, how many mice will you find at Location A?

$\vec{x}_0=\begin{bmatrix}.25\\.35\\.40\end{bmatrix}$ $P=\begin{bmatrix}.75&.20&.30\\.10&.50&.30\\.15&.30&.40\end{bmatrix}$

$P-I=\begin{bmatrix}-.25&.20&.30\\.10&-.50&.30\\.15&.30&-.60\end{bmatrix}\xrightarrow{\text{rref}}\begin{bmatrix}1&0&-2\\0&1&-1\\0&0&0\end{bmatrix}$ $\begin{matrix}x_1=2x_3\\x_2=x_3\\x_3=x_3\end{matrix}\rightarrow\begin{bmatrix}2\\1\\1\end{bmatrix}$

$2+1+1=4$ $\vec{q}=\begin{bmatrix}1/2\\1/4\\1/4\end{bmatrix}=\begin{bmatrix}.5\\.25\\.25\end{bmatrix}$

50% will be at location A, or 50 mice.