Instructions: Show all work. Use exact answers unless asked to round.

1. Solve the differential equation $y'' + 4y' + 4y = 13e^{-2t}$ using the method of variation of parameters.

$$r^2 + 4r + 4 = 0$$

$$(r+2)^2 = 0$$

$$r = -2$$

$$y_1 = e^{-2t}$$

$$y_2 = te^{-2t}$$

$$W = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & e^{-2t} - 2te^{-2t} \end{vmatrix}$$

$$= e^{-4t} - 2te^{-4t} + 2te^{-4t} = e^{-4t}$$

$$y_p = -y_1 \int \frac{y_2 g}{W} + y_2 \int \frac{y_1 g}{W}$$

$$y_p = -e^{-2t} \int \frac{te^{-2t} \cdot 13e^{-2t}}{e^{-4t}} dt + te^{-2t} \int \frac{e^{-2t} \cdot 13e^{-2t}}{e^{-4t}} dt =$$

$$-e^{-2t} \int 13t \, dt + te^{-2t} \int 13 \, dt$$

$$-e^{-2t} \frac{13t^2}{2} + te^{-2t} \cdot 13t$$

$$-\frac{13}{2}t^2 e^{-2t} + 13t^2 e^{-2t} = \frac{13}{2}t^2 e^{-2t}$$

$$y = Ae^{-2t} + Bte^{-2t} + \frac{13}{2}t^2 e^{-2t}$$