Name _____ KEY _____

Math 255, Quiz #4, Summer 2012

Instructions: Show all work. Use exact answers except in word problems or when specifically asked to round.

1. Solve the homogeneous equation $\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$.

$$y = vx$$
$$y' = v'x + v$$
$$v = \frac{y}{x}$$

$$\frac{x - vx}{x + vx} = \frac{1-v}{1+v} - v = \frac{(1-v)}{(1+v)} v'x$$

$$\frac{1-v-v-v^2}{1+v} = v'x$$

$$\frac{1-2v-v^2}{1+v} = v'x$$

$$\frac{dv \,(1+v)}{-1+2v+v^2} = -\frac{1}{x}\,dx \qquad \int \frac{\frac{1}{2}du}{u} = \boxed{\frac{1}{2}\ln|v^2+2v-1| = -\ln x + C}$$

$$u = v^2 + 2v - 1$$
$$\frac{du}{dv} \; 2v^2 + 2 = 2(v+1)$$

$$\boxed{\frac{1}{2}\ln\left|\frac{y^2}{x^2} - 2\frac{y}{x} - 1\right| = -\ln|x| + C}$$

2. Determine if the equation $6xy\,dx + (4y^2 + 9x^2)dy = 0$ is exact (as it's written). If it is not exact, determine if an integrating factor exists that will make it exact. If such a factor exists, or if it is already exact, solve the equation.

$$6x \neq 18x$$

$$\frac{18x - 6x}{6xy} = \frac{12x}{6xy} = \frac{2}{y}$$

$$e^{\int \frac{2}{y} dy} = e^{\ln y^2} = y^2$$

$$6xy^3 dx + \left(4y^4 + 9x^2y^2\right) dy$$
$$18xy^2 = 18xy^2$$

$$\int 6xy^3 \, dx$$
$$3x^2y^3 + f(y)$$

$$\int 4y^4 + 9x^2y^2 \, dy$$
$$\frac{4}{5}y^5 + 3x^2y^3 + g(x)$$

$$\boxed{\frac{4}{5}y^5 + 3x^2y^3 = C}$$