Name KEY

Math 255, Quiz #11, Summer 2012

Instructions: Show all work.

1. Verify that the Laplace transform of $\mathcal{L}[x^3] = \frac{6}{s^4}$ using the definition.

$$\int_0^{\infty} e^{-st} t^3 \, dt =$$

	u	dv
$+$	t^3	e^{-st}
$-$	$3t^2$	$-\frac{1}{s}e^{-st}$
$+$	$6t$	$\frac{1}{s^2}e^{-st}$
$-$	6	$\frac{-1}{s^3}e^{-st}$
	0	$\frac{1}{s^4}e^{-st}$

$$\frac{-1}{s}t^3e^{-st} - 3t^2\frac{1}{s^2}e^{-st} + -6t\frac{1}{s^3}e^{-st} - 6\frac{1}{s^4}e^{-st}\Big|_0^{\infty}$$

$$-0-0-0-0+0+0+0+\frac{6}{s^4}(1)$$

$$= \boxed{\frac{6}{s^4}}$$

2. Solve the differential equation $y'' - 5y' + 6y = e^t$, y(0)=0, y'(0)=0 using Laplace transforms.

$$s^2Y(s) - 5sY(s) + 6Y(s) = \frac{1}{s-1}$$

$$(s^2-5s+6)Y(s) = \frac{1}{s-1}$$

$$Y(s) = \frac{1}{(s-1)(s-3)(s-2)} = \frac{A}{s-1} + \frac{B}{s-3} + \frac{C}{s-2}$$

$$1 = A(s-3)(s-2) + B(s-1)(s-2) + C(s-1)(s-3)$$

$s=1$ $\quad 1 = A(-2)(-1) \Rightarrow 1=2A \quad A = \frac{1}{2}$

$s=2$ $\quad 1 = C(-1)(1) \quad C=-1$

$s=3$ $\quad 1 = B(2)(1) \quad B = \frac{1}{2}$

$$Y(s) = \frac{1}{2}\frac{1}{s-1} + \frac{1}{2}\frac{1}{s-3} + (-1)\frac{1}{s-2}$$

$$\boxed{y(t) = \frac{1}{2}e^t + \frac{1}{2}e^{3t} - e^{2t}}$$