Name KEY

Math 255, Quiz #10, Spring 2012

Instructions: Show all work. Find the series solution (or solutions) to the differential equation given below. List the first 5 terms in each series.

1. $x^2y'' + 4x^3y' + 2y = 0$, centered at the regular singular point $x_0 = 0$.

$$y = \sum_{n=0}^{\infty} c_n (x^{n+r}) \qquad y' = \sum_{n=1}^{\infty} c_n (n+r) x^{n+r-1} \qquad y'' = \sum_{n=2}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2}$$

$$x^2 \sum_{n=2}^{\infty} c_n (n+r)(n+r-1) x^{n+r-2} + 4x^3 \sum_{n=1}^{\infty} c_n (n+r) x^{n+r-1} + 2 \sum_{n=0}^{\infty} c_n x^{n+r} = 0$$

$$\sum_{n=2}^{\infty} c_n (n+r)(n+r-1) x^{n+r} + \sum_{n=1}^{\infty} 4 c_n (n+r) x^{n+r+2} + \sum_{n=0}^{\infty} 2 c_n x^{n+r} = 0$$

$$\sum_{n=2}^{\infty} c_n (n+r)(n+r-1) x^{n+r} + \sum_{n=3}^{\infty} 4 c_{n-2} (n+r-2) x^{n+r} + \sum_{n=0}^{\infty} 2 c_n x^{n+r} = 0$$

$n=2$ $\qquad$ $n=3$ $\qquad$ $n=0$, $n=1$, $n=2$

$$x^r \left[\sum_{n=2}^{\infty} c_n (n+r)(n+r-1) x^n + \sum_{n=3}^{\infty} 4 c_{n-2} (n+r-2) x^n + \sum_{n=0}^{\infty} 2 c_n x^n \right] = 0$$

$$c_2 (2+r)(r+1) x^2 + \sum_{n=3}^{\infty} c_n (n+r)(n+r-1) x^n + \sum_{n=3}^{\infty} 4 c_{n-2} (n+r-2) x^n +$$

$$2c_0 x^0 + 2c_1 x + 2c_2 x^2 + \sum_{n=3}^{\infty} 2 c_n x^n = 0$$

$$2c_0 + 2c_1 x + [c_2 (2+r)(r+1) + 2c_2] x^2 + \sum_{n=3}^{\infty} x^n \left[c_n (n+r)(n+r-1) + 4c_{n-2}(n+r-2) + 2c_n \right] = 0$$

$2c_0 = 0$

$c_0 = 0$

$2c_1 = 0$

$c_1 = 0$

$c_2 (2+r)(r+1) + 2c_2 = 0$

~~$c_2 = 0$ or~~

$(2+r)(r+1) + 2 = 0$

$r^2 + 3r + 2 + 2 = 0$

$r^2 + 3r + 4 = 0$

$r = \dfrac{-3 \pm \sqrt{9-16}}{2}$ = imaginary

$c_2 = 0$

$c_n [(n+r)(n+r-1) + 2] = -4 c_{n-2} (n+r-2)$

$$c_n = \frac{-4 c_{n-2} (n+r-2)}{(n+r)(n+r-1) + 2}$$

lowest coeff $c_{n-2} = c_3$