Name KEY

Math 254, Quiz #9, Summer 2012

Instructions: Show all work. Use exact answers unless asked to round.

1. Find the center of mass for the region bounded by the graphs $y = \sqrt{x}, y = 0, x = 4$ with the density function $\rho = kxy$. Give your final answer as a coordinate point.

$$M_y = \iint_R x\,\rho(x,y)\,dA = \int_0^4\int_0^{\sqrt{x}} kx^2 y\,dy\,dx = \int_0^4 kx^2\,\frac{y^2}{2}\Big|_0^{\sqrt{x}}\,dx = \int_0^4 \frac{k}{2}x^2\cdot x\,dx =$$

$$\frac{k}{2}\int_0^4 x^3\,dx = \frac{k}{2}\left[\frac{x^4}{4}\right]_0^4 = \frac{k}{8}\cdot 256 = 32k$$

$$M_x = \iint_R y\,\rho(x,y)\,dA = \int_0^4\int_0^{\sqrt{x}} kxy^2\,dy\,dx = \int_0^4 kx\,\frac{y^3}{3}\Big|_0^{\sqrt{x}}\,dx = \int_0^4 \frac{k}{3}x\cdot x^{3/2}\,dx$$

$$= \frac{k}{3}\int_0^4 x^{5/2}\,dx = \frac{k}{3}\left[\frac{2}{7}x^{7/2}\right]_0^4 = \frac{2k}{21}\cdot 128 = \frac{256k}{21}$$

$$M = \iint_R \rho(x,y)\,dA = \int_0^4\int_0^{\sqrt{x}} kxy\,dy\,dx = \int_0^4 kx\,\frac{y^2}{2}\Big|_0^{\sqrt{x}}\,dx = \int_0^4 kx\cdot\frac{x}{2}\,dx$$

$$= \frac{k}{2}\int_0^4 x^2\,dx = \frac{k}{2}\left[\frac{x^3}{3}\right]_0^4 = \frac{k}{2}\cdot\frac{64}{3} = \frac{32k}{3}$$

$$\bar{x} = \frac{M_y}{M} = \frac{32k}{1}\cdot\frac{3}{32k} = 3$$

$$\bar{y} = \frac{M_x}{M} = \frac{256k}{21}\cdot\frac{3}{32k} = \frac{8}{7}$$

$$\boxed{\left(3, \frac{8}{7}\right)}$$