Name KEY

Math 254, Quiz #2, Summer 2012

Instructions: Show all work. Give answers in exact form unless the problem begins with decimals.

1. Find the velocity, speed, acceleration and jerk of a particle traveling along the path $\vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, (\pi, 2)$. Evaluate each derivative at the specified point.

$r'(t) = (1-\cos t)\hat{i} + \sin t\,\hat{j}$ $\quad t - \sin t = \pi$ when $t = \pi$

$r''(t) = \sin t\,\hat{i} + \cos t\,\hat{j}$ $\quad 1 - \cos t = 2$ when $t = \pi$

$r'''(t) = \cos t\,\hat{i} - \sin t\,\hat{j}$

$\|r'(t)\| = \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} = \sqrt{1 - 2\cos t + 1} = \sqrt{2 - 2\cos t}$

$r'(\pi) = (1 - (-1))\hat{i} + 0\hat{j} = 2\hat{i}$ $\quad \|r'(\pi)\| = \sqrt{2 - 2(-1)} = \sqrt{4} = 2$

$r''(\pi) = 0\hat{i} + -1\hat{j} = -\hat{j}$

$r'''(\pi) = -1\hat{i} - 0\hat{j} = -\hat{i}$

2. Given the acceleration function $\vec{a}(t) = \sin t\vec{i} - \vec{j} + t\vec{k}$, and initial conditions $\vec{v}(0) = 2\vec{j} + \vec{k}, \vec{r}(1) = 4\vec{i}$, find the velocity and position functions.

$\int a(t) = (-\cos t + C_1)\hat{i} - (t + C_2)\hat{j} + \left(\frac{t^2}{2} + C_3\right)\hat{k}$

$-\cos 0 + C_1 = 0$ $\quad -0 + C_2 = 2$ $\quad 0 + C_3 = 1$

$C_1 = 1$ $\quad C_2 = 2$ $\quad C_3 = 1$

$v(t) = (-\cos t + 1)\hat{i} + (t + 2)\hat{j} + \left(\frac{t^2}{2} + 1\right)\hat{k}$

$\int v(t) =$

$\left(-\sin t + t + C_1\right)\hat{i} + \left(-\frac{t^2}{2} + 2t + C_2\right)\hat{j} + \left(\frac{t^3}{6} + t + C_3\right)\hat{k}$

$-\sin(1) + (1) + C_1 = 4$ $\quad -\frac{1}{2} + 2 + C_2 = 0$ $\quad \frac{1}{6} + 1 + C_3 = 0$

$C_1 = 3 + \sin(1)$ $\quad -\frac{3}{2} = C_2$ $\quad -\frac{7}{6} = C_3$

$r(t) = \left[-\sin t + t + 3 + \sin(1)\right]\hat{i} + \left(-\frac{t^2}{2} + 2t - \frac{3}{2}\right)\hat{j} + \left(\frac{t^3}{6} + t - \frac{7}{6}\right)\hat{k}$